OPTIMIZATION OF THE LANCZOS KERNEL PARAMETER FOR INTERPOLATION IMAGE

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Abstract

In this paper, the parameter optimization of the 1P Lanczos interpolation kernel is performed. The optimal value of the parameter was determined experimentally, by interpolating some test images. The accuracy of the Lanczos interpolation kernel was measured using MSE. For the purpose of comparative analysis interpolation was performed the test images with the implementation of the 1P Keys and Dodgson kernels. The results are presented graphically and tabularly.

Keywords: convolution, interpolation, parametric convolution interpolation kernel, optimal value parameter.

1. INTRODUCTION

The interpolation theory has been drawing attention since the early days. The word 'interpolation' originates from Latin verb 'interpolare' and means 'inserting new members between given members'. In English literature, the word 'interpolation' is first encountered and used since 1612, and it's used in sense of exchanging (text) after inserting a new one. Newton’s geometrical curve which pass through any given points descriptions, 1675 are the very basis of development of the interpolational methods. During the 19th Century, the interpolation methods based on polynomials had been intensively studied. Schöneberg 1946 defined basis functions which enable interpolation of equidistant data [1]. During 1970 interpolation techniques based on cubic convolution had been further developed [2], [3]. Researches about convolution interpolation are involved in many works [4] – [8]. Convolution interpolation is realised with the appliance of interpolation convolution kernel. Different interpolation kernels enable different accuracy and efficacy of interpolation algorithms. Velocity of interpolation algorithms execution and their numerical accuracy are directly linked with choice of interpolation kernel. In order to increase the estimate of the interpolated value, the parameterization of the kernel was performed. The kernel parameter can be determined according to some criteria so as to obtain the highest accuracy of the estimate, ie to minimize the interpolation error. Thus, the determined value of the kernel parameter becomes optimal.

In this paper, the optimal value of one-parameter (1P) Lanczos kernel is decided by experimental ways, at interpolation of some test images. For each test image, the optimal value of kernel parameter, \( \alpha_{\text{opt.}} \) is determined.

For the purpose of comparative analysis, interpolation was performed in test images using the 1P Keys interpolation kernel [4] and the Dodgson kernel [5]. The accuracy of the interpolation kernel was measured using the mean square error (MSE) between the accurate and the interpolated value.

In the following, 1P Lanczos interpolation kernel, which is based on the sinc function, is described. An experiment and an algorithm are described by which the optimal value of the kernel parameter is determined. The results are presented using tables and graphs.

2. LANCZOS INTERPOLATION KERNEL

1P Lanczos interpolation kernel is defined with[9]:

\[
L(x) = \begin{cases} 
\frac{\sin(\pi x)}{\pi x} & , \quad 0 \leq |x| < \alpha \\
\frac{\sin((\pi x)/\alpha)}{(\pi x)/\alpha} & , \quad |x| \geq \alpha 
\end{cases}
\]

(1)
where is α kernel parameter.

In Fig. 1 1P Lanczos interpolation kernel is shown for some parameter values α.

**Fig. 1.** Lanczos interpolation kernel for different parameter value α.

![Lanczos interpolation kernel](image)

**3. EXPERIMENTAL RESULTS AND ANALYSIS**

The optimal parameter value α and the minimal mean square error of interpolation is determined by experimental ways at image interpolation case is determined.

**3.1. The Experiment**

Algorithm for determining optimal value of kernel parameter consists of following steps:

**Input:** Test image X dimension MxN, kernel r, length L, αmin, αmax, Δα.

**Output:** αopt.

**Step 1:** Construction one-dimensional sequence xMN by connecting rows of the matrix X.

**FOR** α = αmin: Δα; αmax

**Step 2:** Construction of the kernel r in function α.

**FOR** i = 1 : M·N - (L+2)

**Step 3:** Estimation ˆx(i+L-1) by applying convolution with Lanczos kernel.

\[
\hat{x}(i+L-1) = \left[ x_{i} x_{i-L-2}, x_{i-L}, x_{i-L+2} \right] \otimes r_{a}
\]

**Step 4:** Estimation the interpolation error:

\[
e_{a,i} = x(i+L-1) - \hat{x}(i+L-1)
\]

**END i**

**Step 5:** Determination of the mean square error,

\[
MSE_{a} = \frac{1}{MN-(L+2)} \sum_{i=1}^{MN-(L+2)} |e_{a,i}|^{2}
\]

**END α**

**Step 6:** Determination optimal value, αopt of kernel parameter:

\[α_{opt} = \arg \min_{α} (MSE_{a}).\]

**3.2 The Base**

The base consists of K = 8 standard test images: Lena, Pappers, Goldhill, Camerman, Boats, Barbara, Baboon, Cat.
3.3 Results

By Lanczos kernel application at interpolating some test images, the results were obtained for $\alpha_{\text{opt}}$ and $MSE_{\text{min}}$ which are shown in Table 1 and in Fig. 4. For comparative analysis, the interpolation is done on test images also by using 1P Keys kernel and quadratic Dodgson kernel. The results for $\alpha_{\text{opt}}$ and $MSE_{\text{min}}$ are shown in Table 2 and Table 3. Mean value of all optimal parameters and minimal values of MSE are shown in all tables.

**Table 1. Minimal mean square error for Lanczos kernel for test images from the base.**

<table>
<thead>
<tr>
<th>Image</th>
<th>$\alpha_{\text{opt}}$</th>
<th>$MSE_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>2.2200</td>
<td>26.5959</td>
</tr>
<tr>
<td>Peppers</td>
<td>2.2040</td>
<td>98.6184</td>
</tr>
<tr>
<td>Goldhill</td>
<td>1.3370</td>
<td>35.7681</td>
</tr>
<tr>
<td>Cameraman</td>
<td>1.3370</td>
<td>347.4825</td>
</tr>
<tr>
<td>Boats</td>
<td>1.3380</td>
<td>37.7240</td>
</tr>
<tr>
<td>Barbara</td>
<td>1.3320</td>
<td>91.5108</td>
</tr>
<tr>
<td>Baboon</td>
<td>1.3320</td>
<td>74.4397</td>
</tr>
</tbody>
</table>

**Table 2. Minimal mean square error for 1P Keys kernel for test images from the base.**

<table>
<thead>
<tr>
<th>Image</th>
<th>$\alpha_{\text{opt}}$</th>
<th>$MSE_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>-0.7000</td>
<td>26.5724</td>
</tr>
<tr>
<td>Peppers</td>
<td>-0.4000</td>
<td>97.0203</td>
</tr>
<tr>
<td>Goldhill</td>
<td>0</td>
<td>35.7683</td>
</tr>
<tr>
<td>Cameraman</td>
<td>-0.1000</td>
<td>347.1408</td>
</tr>
<tr>
<td>Boats</td>
<td>-0.1000</td>
<td>37.6834</td>
</tr>
<tr>
<td>Barbara</td>
<td>-0.5000</td>
<td>31.6406</td>
</tr>
<tr>
<td>Baboon</td>
<td>0.9000</td>
<td>86.6896</td>
</tr>
<tr>
<td>Cat</td>
<td>-0.2000</td>
<td>73.9280</td>
</tr>
</tbody>
</table>

**Table 3. Minimal mean square error for Dodgson for test images from the base.**

<table>
<thead>
<tr>
<th>Image</th>
<th>$\alpha_{\text{opt}}$</th>
<th>$MSE_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>-2.7000</td>
<td>33.7984</td>
</tr>
<tr>
<td>Peppers</td>
<td>2.7000</td>
<td>100.1820</td>
</tr>
<tr>
<td>Goldhill</td>
<td>1.1000</td>
<td>35.7683</td>
</tr>
<tr>
<td>Cameraman</td>
<td>-1.6000</td>
<td>347.4828</td>
</tr>
<tr>
<td>Boats</td>
<td>-2.7000</td>
<td>37.7242</td>
</tr>
<tr>
<td>Barbara</td>
<td>1.1000</td>
<td>35.1087</td>
</tr>
<tr>
<td>Baboon</td>
<td>-1.7000</td>
<td>91.5440</td>
</tr>
<tr>
<td>Cat</td>
<td>0.8000</td>
<td>74.4423</td>
</tr>
</tbody>
</table>

By conducting statistics analysis of parameter optimal values from Tbl. 1, mean value $\mu$ and variance $\sigma^2$ is determined. On the basis of $\mu$ and $\sigma^2$, Gauss normal function, $p(\alpha)$ is determined and shown on the Fig. 5.
3.4 Comparative analysis

Based on the results shown on Fig. 2 and Table 1, Table 2 and Table 3, it is concluded that:

a) Range of optimal parameter values of the Lanczos kernel calculated for test images (Tbl. 1) $\alpha_{opt} \in [1.332 – 2.22]$ and mean value $\alpha_{opt} = 1.6639$.

b) Range of optimal parameter values of the Keys kernel calculated for test images (Tbl. 2) $\alpha_{opt} \in [-0.7 - 0]$ and mean value $\alpha_{opt} = -0.1375$.

c) Range of optimal parameter values of the Dodgson kernel calculated for test images (Tbl. 3) $\alpha_{opt} \in [-2.7 – 1.1]$ and mean value $\alpha_{opt} = -0.1375$.

d) By the comparison of MSE, it can be concluded that the MSE obtained by interpolation using the 1P Lanczos kernel in relation to the MSE obtained by interpolation using the 1P Keys kernel, is $\frac{MSE_{min\_L}}{MSE_{min\_K}} = 92.9918 / 92.055 = 1.0102$ times greater.

e) By the comparison of MSE, it can be concluded that the MSE obtained by interpolation using the 1P Lanczos kernel in relation to the MSE obtained by interpolation using the Dodgson kernel is $\frac{MSE_{min\_L}}{MSE_{min\_D}} = 92.9918 / 94.536 = 0.984$ times lesser.

4. CONCLUSION

In this paper, the results of application 1P Lanczos kernel at image interpolation were shown. The optimal value of kernel parameter is defined by experimental ways $\alpha_{opt} = 1.6639$. Kernel efficiency is calculated by MSE. By the analysis of MSE it can be concluded that the 1P Lanczos kernel is more efficient than Dodgson kernel, while it is less efficient than 1P Keys kernel.

REFERENCES


