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NEW KIND OF DIGITAL FILTER BASED ON THE FREQUENCY LOCKED LOOP

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Abstract

In this article one new kind of FIR (Finite Impulse Response) digital filter is described. The filter is intended for the filtering of pulse periods. The filter is designed using special kind of the Frequency Locked Loop (FLL) of the fifth order, which is based on the measurement and processing of the input periods only. The FLL is described by the difference equations. Z transform approach was used for the system analysis. The procedure and methodology of the digital filter design is described in details. The design is performed using Mat-lab tools, dedicated to the design of the FIR digital filter. Analyzes of the filter were performed in the time and frequency domains. Computer simulation of the system in time domain is made to enable precise insight into the system functioning and to proof the correctness of the mathematical analyzes.

Keywords: Digital circuits, Digital filters, PLL, FLL, Pulse circuits.

INTRODUCTION

The idea of generating new types of PLLs and FLLs by processing the input and output signal periods and the time differences between them, has fully met expectations. Let us call these new systems, in short, the period FLL and the period PLL. In the papers [1] to [11], the period PLLs and FLLs were applied in the field of frequency averaging, phase shifting, time shifting, phase control, tracking, predicting, frequency synthesizers, noise rejection, frequency multipliers and the others. These works have shown that the field of PLLs and FLLs application has significantly increased, compared to the classical PLLs and FLLs.

This article discovered one new special property of the period FLL. For the appropriate choice of the system parameters, the period FLL can function as FIR digital filter intended for the filtering of pulse signal periods. That was possible due to the fact that the difference equations of the period FLLs is structurally very similar to FIR digital filter equations. This article demonstrates how we can adapt the transfer function of the fifth order period FLL and choose its system parameters using FIR digital filter theory, in order to function as the desired digital filter, intended for the filtering of pulse periods. To do that it was necessary first to define the difference equations of the period FLL of the fifth order and to find out its appropriate transfer functions.

The rest of articles and books [12] to [17] in the references, are used as the theoretical base for the different fields which are used in these analyzes, for electronics implementations and for the development necessities.

TRANSFER FUNCTIONS OF FLL

Let us first consider Fig. 1, which represents a general case of an input signal Sin and an output signal Sop of the period FLL of the fifth order. The periods TI_k and TO_k, as well as the time difference τ_k, occur at discrete times t_k, t_{k+1}, t_{k+2}, t_{k+3}, t_{k+4}, t_{k+5}…, which are defined by the falling edges of the pulses of Sop in Fig. 1. In the real time applications of the period FLL, any output period can be calculated only using the previous input periods of the lower order. Taking this fact in account, the general difference equation which describes period FLL of the fifth order, intended for the real time applications, is presented in eq. (1), where b_1, b_2, b_3, b_4 and b_5 are the system parameters. Because of simplicity, all expressions for periods TO and TI in eq. (1), instead of discrete times in brackets e.g. TO(k+4), they use the corresponding index TO_{k+4}. Because of necessity for the precise recognition of the FLL state at any discrete time, one additional difference equation, describing the time difference τ, is presented in eq. (2). It comes out as the natural relation between the variables in Fig. 1. To determine the transfer functions of the period FLL, the Z transform of eqs. (1) and (2) are presented in eqs. (3) and (4), respectively, where TO_0, TI_0, in eq. (3) and τ_0 in eq. (4) represent the initial conditions of variables TO_k, TI_k and τ_k. Because of simplicity, expressions TO(z) and TI(z) are denoted by TO_z and TI_z in eq. (3). Since the process start at $t=t_k$, all values of variables, before $t=t_k$, are equal to zeros. Therefore, according to eq. (1), $TO_1=b_1 \cdot TI_0$, $TO_2=b_1 \cdot TI_1+b_2 \cdot TI_0$, $TO_3=b_1 \cdot TI_2+ b_2 \cdot TI_1+b_3 \cdot TI_0$ and $TO_4=b_1 \cdot TI_3+b_2 \cdot TI_2+b_3 \cdot TI_1 +b_4 \cdot TI_0$. Entering TO_1, TO_2, TO_3 and TO_4 into eq. (3), TO(*z*) is calculated and shown in eq. (5).

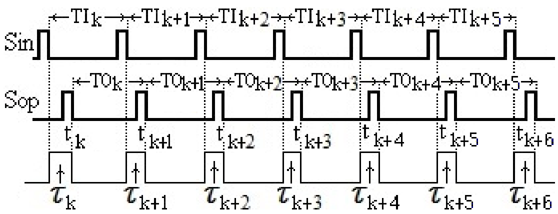

Fig. 1. *The time relations between the input and output variables of period FLL.*

$$TO_{k+5} = b_1 TI_{k+4} + b_2 TI_{k+3} + b_3 TI_{k+2} + b_4 TI_{k+1} + b_5 TI_k \quad (1)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \quad (2)$$

$$\begin{aligned} & z^5 TO_z - z^5 TO_0 - z^4 TO_1 - z^3 TO_2 - z^2 TO_3 - z TO_4 = \\ & b_1 (z^4 TI_z - z^4 TI_0 - z^3 TI_1 - z^2 TI_2 - z TI_3) + b_2 (z^3 TI_z - \\ & - z^3 TI_0 - z^2 TI_1 - z TI_2) + b_3 (z^2 TI_z - z^2 TI_0 - z TI_1) + \\ & + b_4 (z TI_z - z TI_0) + b_5 TI_z \end{aligned} \quad (3)$$

$$z\tau(z) - z\tau_0 = \tau(z) + TO(z) - TI(z) \quad (4)$$

$$TO(z) = TI(z) \frac{z^4 b_1 + z^3 b_2 + z^2 b_3 + z b_4 + b_5}{z^5} + TO_0 \quad (5)$$

It is now necessary to investigate under which conditions the circuit described possesses the properties of a FLL. To do that, let us suppose that the step input is TI(*k*) = TI = constant. Substituting the Z transform of TI(*k*) i.e. TI(*z*) = TI· *z*/(*z*-1) into eq. (5) and using the final value theorem, it is possible to find the final value of the output period TO_∞, which the FLL reaches in stable state. We can calculate TO_∞ = lim TO(*k*) if $k \to \infty$, using TO(*z*). This is shown in eq. (6). It comes out from eq. (6), that TO_∞=TI if eq. (7) is satisfied. This means that the system described possesses the properties either of a FLL or a PLL. To make decision, it is necessary to determine the behaviour of time difference "τ". Entering TO(*z*) from eq. (5), into eq. (4) and taking in account eq. (7), τ(*z*) is calculated and presented in eq. (8). Substituting now TI(*z*) = TI·*z*/(*z*-1) into eq. (8) and using the final value theorem, it is possible to find the final value of the time difference τ_∞ = lim τ(*k*) if $k \to \infty$, using τ(*z*), given by eq. 8. Taking in account, according to eq. (7), that $-b_5=b_4+ b_3+b_2+b_1-1$, τ_∞ was found and shown in eq. (9). Equation (9) also confirms that the described system possesses the properties of a FLL, since τ_∞ depends on the initial conditions. It comes out that the system does not possess the properties of a PLL.

$$TO_\infty = \lim \Big[\big(z-1\big) TO(z) \Big]_{z \to 1} = TI(b_1 + b_2 + b_3 + b_4 + b_5) \quad (6)$$

$$b_1 + b_2 + b_3 + b_4 + b_5 = 1 \quad (7)$$

$$\begin{aligned} & \tau(z) = TI(z) \cdot [-z^4 + z^3 (b_1 - 1) + z^2 (b_2 + b_1 - 1) + \\ & z(b_3 + b_2 + b_1 - 1) - b_5] / z^5 + (TO_0 + z\tau_0) / (z-1) \end{aligned} \quad (8)$$

$$\begin{aligned} & \tau_\infty = \lim \Big[\big(z-1\big) \tau(z) \Big]_{z \to 1} = \\ & = TI(4b_1 + 3b_2 + 2b_3 + b_4 - 5) + TO_0 + \tau_0 \end{aligned} \quad (9)$$

There are two transfer functions which describe this period FLL. These are $H_{TO}(z)=TO(z)/TI(z)$ and $H_\tau(z)=\tau(z)/TI(z)$ shown in eqs. (10) and (11). They originate from eqs. (5) and (8).

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{z^4 b_1 + z^3 b_2 + z^2 b_3 + z b_4 + b_5}{z^5} \quad (10)$$

$$\begin{aligned} & H_\tau(z) = \frac{\tau(z)}{TI(z)} = [-z^4 + z^3 (b_1 - 1) + \\ & z^2 (b_2 + b_1 - 1) + z(b_3 + b_2 + b_1 - 1) - b_5] / z^5 \end{aligned} \quad (11)$$

design of digital filter

There are important differences between the FIR digital filter and the described FLL. First, the FIR digital filter processes the digitized amplitudes and the FLL processes the digitized periods. Second, the sampling of the input signal is performed at constant time intervals, while the processing of the period may be non-uniform, when the input period is not constant. Since the periods of the input and output signals are time variables, they are distributed in time and they overlap each other, disabling the calculation of the output period with the input period of the same order. Note that the periods of the same order represent the periods defined by the same discrete time. FIR digital filters do not have this phenomenon. Because of this appearance it was necessary to introduce in math of the period FLL new variable, time difference between the input and output, due to which we could control the overlapping and determine the relation between the output and input precisely, as it was shown in the previous chapter. However, there are also very important similarities between these two systems. Both systems are described by difference equations. This FLL, as said, processes only the periods of the input signal and FIR digital filters process only input samples. The difference equations of FLL have a structural form that is very similar to the structural form of the difference equation of the FIR digital filter. The only difference in their equations stems from the already mentioned fact that only lower order input periods can be used in the calculation of the output period, which is not the case with the FIR digital filter. But even so, theory and Mat-lab tools of FIR digital filter can be used effectively to design FIR digital filters based on FLL, which are intended for digital filtering of pulse signal periods.

To demonstrate the adaptation of the period FLL in order to work as the digital filter for the pulse periods, let us first design low pass FIR digital filter of the forth order (N=4), with the sampling frequency fs=10000 Hz and cutoff frequency fg=1600 Hz. If we use triangle windowing and Mat-lab command "fir1", we can get vector "b" of the filter coefficients as b=fir1(N, fn, triang(N+1)), where the normalized cutoff frequency fn=fg/(fs/2). This command gives the next coefficients for FIR digital filters: b_{0d}=0.0620, b_{1d}=0.2314, b_{2d}= 0.4132, b_{3d}=0.2314 and b_{4d}=0.0620 where suffix "d" signifies that these coefficients belong to the digital filter. The corresponding transfer function, for the digital filter described, is $H_d(z) = b_{0d}+b_{1d}\cdot z^{-1} +b_{2d}\cdot z^{-2} +b_{3d}\cdot z^{-3}+b_{4d}\cdot z^{-4}$, or it can be given as in eq. (12). If we compare the transfer function $H_{TO}(z)$ of FLL, given by eq. (10), and eq. (12), we can notice that both of them possess five coefficients or parameters. If we define the parameters of the FLL as: $b_1=b_{0d}$, $b_2=b_{1d}$, $b_3=b_{2d}$, $b_4=b_{3d}$ and $b_5=b_{4d}$, we will get a new expression for H_{TO}, given in eq. (13). Note that the sum of the chosen parameters of FLL $b_{0d}+b_{1d}+b_{2d}+b_{3d}+b_{4d}=1$, satisfies eq. (7). It means that the period FLL is the stable system and that it possesses the same zeros as digital filter. The difference between $H_d(z)$ and $H_{TO}(z)$, shown in eqs. (12) and (13), is in theirs denominators. Namely, their relation can be expressed as $H_{TO}(z)= H_d(z)\cdot z^{-1}$. This means that the frequency responses of $H_{TO}(z)$ and $H_d(z)$ will be almost the same. More precisely, theirs magnitudes will be the same, but the period FLL

$$H_d(z) = \frac{TO(z)}{TI(z)} = \frac{z^4 b_{0d} + z^3 b_{1d} + z^2 b_{2d} + z b_{3d} + b_{4d}}{z^4} \qquad (12)$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{z^4 b_{0d} + z^3 b_{1d} + z^2 b_{2d} + z b_{3d} + b_{4d}}{z^5} \qquad (13)$$

will introduce additional delay of -π [rad] at the output signal, in comparison with the phase at the output of digital filter. In accordance with the Mat-lab definitions, the vectors of coefficients b_d and parameters b_{TO}, which correspond to the transfer functions $H_d(z)$ and $H_{TO}(z)$, are shown in eq. (14). These vectors will be used in commands, devoted to the design of FIR digital filters. According to the Mat-lab vector definitions, the relation between them can be expressed as b_{TO}=[0 b_d].

$$\begin{aligned} b_d &= [b_{0d}\ \ b_{1d}\ \ b_{2d}\ \ b_{3d}\ \ b_{4d}] \\ b_{TO} &= [0\ \ b_1\ \ b_2\ \ b_3\ \ b_4\ \ b_5] = [0\ \ b_{0d}\ \ b_{1d}\ \ b_{2d}\ \ b_{3d}\ \ b_{4d}] \end{aligned} \qquad (14)$$

Using commands "freqz (b_{TO}, 1, 1024, fs)" and "freqz (b_d, 1, 1024, fs)", the frequency responses of $H_{TO}(z)$ and $H_d(z)$ are determined and presented in Fig. 2, for the half of the sample rate. It can be seen that the magnitudes of the digital filter and the FLL are identical with the cutoff frequencies of 1600 Hz, and that the phases which two systems introduced into the output signals differ for expected -180°. Both of them are linear, but for the half of the sample rate, the phase of the FLL is -540° and the phase of the digital filter is -360°.

Using simulation, let us now present the effects of the FLL filtering in time domain. The simulation in time domain is to enable better insight into the procedure and physical meaning of the variables described. All discrete values in simulation were merged to form continuous curves. Note that all variables in the following diagram are presented in time units. The time

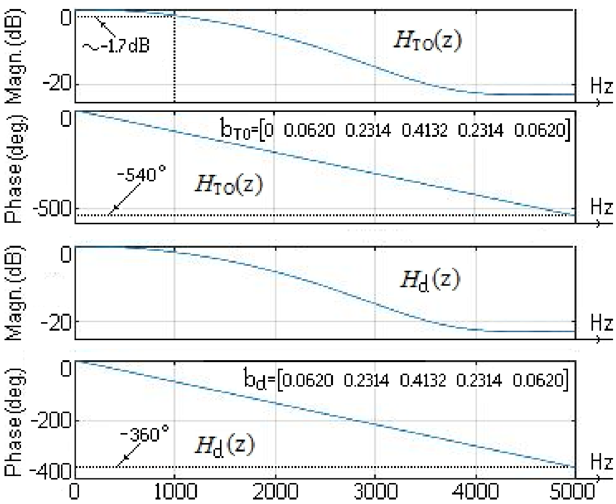

Fig. 2. *Magnitudes and phases of the frequency responses of $H_{TO}(z)$ and $H_d(z)$.*

unit can be, µsec, msec or any other, but assuming the same time units for all input and output time variables. It was more suitable to use just "time unit" or abbreviated "t.u." in the text. It was more convenient to omit the indication „t.u." in diagram. For this purpose the input period $TI(k+1)=10+S_1(k)+S_2(k)$ [t.u.] was fed into the input of the FLL, where: $S_1(k)=5\cdot\sin[2\pi/fs\cdot f_1\cdot k]$, $S_2(k)=5\cdot\sin[2\pi/f_s\cdot f_2\cdot k]$. The frequency f_1=1000 Hz and the frequency f_2=3000 Hz. Since f_s=10000 Hz, it means that S_1 is sampled by 10000 Hz/1000 Hz = 10 samples/period, what is acceptable resolution for this presentation, Fig. 3a. Signal S_2 is sampled with 10000 Hz/3000 Hz ~ 3.33 samples/period. This small number of samples does not provide good presentation of S_2. Because of that, S_2 appeared in form of needles at the input of the FLL, Fig. 3b. The input and output signals TI(k) and TO(k) are shown in Fig. 3c and separated TO(k) is shown in Fig. 3d. The initial conditions are TO_0=0 t.u., τ_0=0 t.u. and TI_0=10 t.u. The described processing of the FLL is shown for the 80 steps in Fig. 3. It can be seen that the FLL filtered and mainly preserved the shape of input signal S_1, whose frequency f_1=1000 Hz is less than cutoff frequency f_c=1600 Hz, but signal S_2 disappeared at the output, shown in Fig. 3d, because f_2 =3000 Hz belongs to the stop band of the FLL.

The impression about the same processing will be completed if we present it in the frequency domain. Using commands "fft" and "stem", frequency spectrums of TI_k and TO_k are presented in the whole sample rate, in Fig. 4. We see in Fig. 4 that the frequency at 3000 Hz, corresponding to S_2, is suppressed, what is in

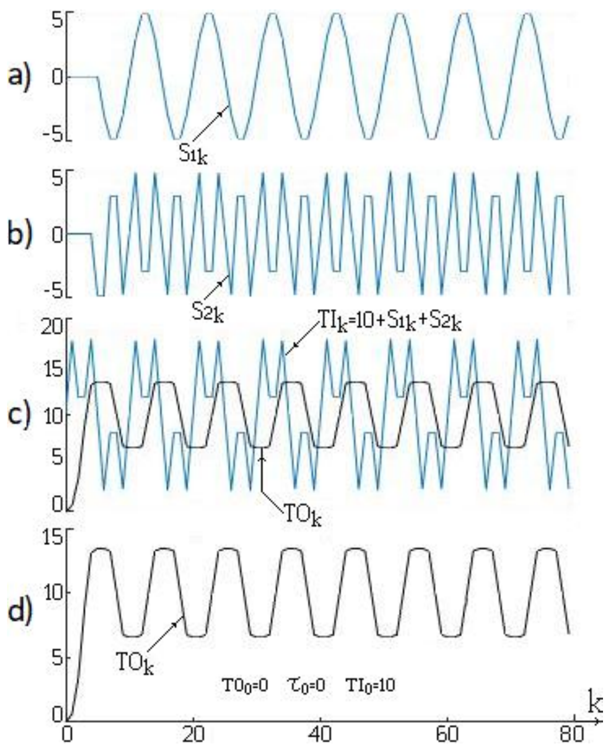

Fig. 3. *Presentations of the initial conditions and signals in time domain:* ***a****. $S_1(k)$,* ***b****. $S_2(k)$,* ***c****. TO(k) and TI(k),* ***d****. TO(k) and the initial conditions.*

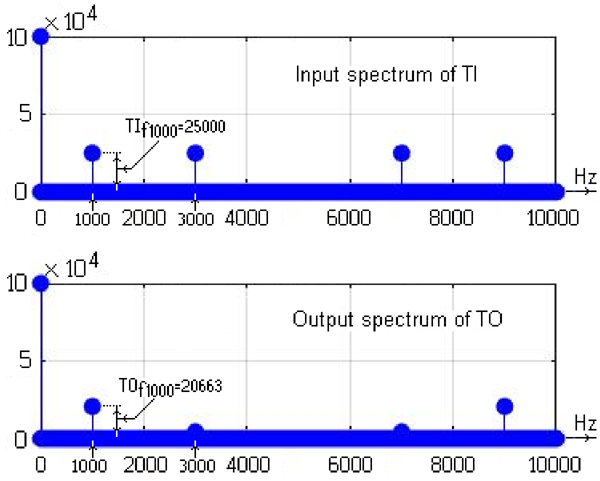

Fig. 4. *Frequency spectrums of the input TI and the output TO*

agreement with Fig. 3d. Namely, the output of FLL, shown in Fig. 3d, changed slightly the form in comparison to S_1, shown in Fig. 3a, but signal S_2 is mainly suppressed. According to the results of the computer listing, shown in Fig. 4, the frequency component at 1000 Hz, corresponding to S_1, is attenuated for 20 log (TO_{f1000}/TI_{f1000}) = 20 log (20663/25000)=-1.65 [db]. The similar result we can see at the magnitude frequency response of $H_{TO}(z)$ in Fig. 2. Namely, if we magnify Fig. 2, using the proportionality, it could be determined roughly, that the attenuation at 1000 Hz is about -1.7 [db], what is close to -1.65 [dB].

CONCLUSION

This article discovered how we can design FIR digital filter for the periods of semi-digital signals, based on the period FLL. For this purpose we use the FIR digital filter theory, the appropriate Mat-lab tools and the theory of the period FLL. The design consists of two steps. In the first step we design a desired FIR digital filter to meet the appropriate requirements and in the second step we generate the transfer function of the period FLL, which is able to cover the zeros of the digital filter. If a digital filter is of the n^{th} order, such kind of adaptation is possible, if the period FLL is of the $(n+1)^{th}$ order.

Digital filters, based on the period FLL, do not require A/D and D/A converters. They are based on the measurement of time instead of amplitude, and therefore they provide better accuracy of measurements and simpler and cheaper electronic solutions. Periodic and non-periodic pulse signals are widely present in electronics, in telecommunications, as well as in control and measurement applications. Therefore, there is an obvious need to filter them in many of these applications.

The results of this article represent the base for the further possible applications of the period FLL in many fields. However, the most likely and useful next steps are the development of the higher order FIR digital filters, as well as the development of the IIR digital filters, based on the period FLL.

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