

## ESTIMATION OF THE BIT ERROR RATE AND QUALITY FACTOR IN OPTICAL SYSTEMS

**Dragan Mitić**

*Institute for  
Telecommunications and  
Electronics,  
IRITEL a.d. BELGRADE,  
Serbia*

**Aleksandar Lebl**

*Institute for  
Telecommunications and  
Electronics,  
IRITEL a.d. BELGRADE,  
Serbia*

**Verica Vasiljević**

*Faculty of  
Information Technology,  
University Slobomir P,  
Bjeljina,  
Bosnia and Herzegovina*

### Abstract

*The purpose of the paper is to define the connection between error parameters in the networks realized using optical transmission system and optical wireless systems. Due to high volume of transmitted data caused by associating the great number of data flows, very high error requests are expected during data transmission. In this paper it is to emphasize measuring procedures, estimate individual error parameters and to explain the relation between them.*

**Keywords:** Bit Error Rate, Quality Factor, Dense Wavelength Division Multiplexing.

### INTRODUCTION

The development in basic networks is performed by completely optical Dense Wavelength Division Multiplexing, DWDM [1], technology on terabit capacity on thousands of kilometres distance. The advantage of DWDM technology is the possibility of efficient recovery and signal amplification, using one amplifier with erbium-doped optical fiber for all channels in the same time, which is suitable for the transmission in the basic network.

Optical devices, which allow such capacity transmission, must cope with a number of physical parameters as signal attenuation, noise, polarization dispersion, nonlinear interference, and so on. The goal of this paper is to define the relation between error parameters in optical networks and optical wireless networks, where optical beam is transmitted through the atmosphere. The connections without optical fiber are sensitive

to the transmission environment state and, comparing to pure optical fiber connections, their quality depends on the time of a day, climate, etc [2].

Chapter 2 explains the definition of bit error rate (BER) and defines the distribution to be used in the future. Chapter 3 defines the estimation of the accuracy of measuring the bit error ratio and determines the approximate required measurement time. The definition of the Q-factor is given in Chapter 4, while an overview of the relationship between BER and Q-Factors is explained in Chapter 5.

### BIT ERROR RATE DEFINITION

One of the main parameters which define the connection quality for data transmission is BER. The quality of different data transmission systems may be compared using BER. The value of BER is expressed by the equation:

$$BER = \frac{N_{ERR}}{N_{BIT}}, \quad (1)$$

where  $N_{ERR}$  is the number of incorrectly transmitted bits and  $N_{BIT}$  is the total number of received bits in the defined time interval [3], [4].

In modern transmission networks information is transmitted in greater blocks, called packets. Each packet consists of a number of bits, which may be selected or prescribed by the network type. A faulty bit transmission causes the whole packet degradation [5]. When considering the error rate, the huge data amount is lost. This error rate is determined by the relation (2).

$$PER = \frac{N_{ERP}}{N_P}, \quad (2)$$

in this equation it is:

$N_{ERP}$  – the transmitted packets number with at least one incorrectly transmitted bit;

$N_P$  – the total transmitted packets number.

The relative time interval  $p$  is defined as the important error parameter for the connections without optical fiber. The value of  $p$  presents the percent of connection disruption  $t$  till the total time  $T_c$  according to the equation (3).

This parameter is based on the possibility of connection failure due to atmospheric turbulence and the received power fluctuation. In order to reliably determine this parameter it is necessary to select a sufficiently long period  $T_c$ . Its duration is usually one year.

$$p = \frac{\sum_{i=1}^{\infty} t_i}{v_i} \quad (3)$$

As it may be seen from (1), it is necessary to know the total number of transmitted bits  $N_{BIT}$ . This number may be determined by continuous monitoring of the transmitted bits number. The BER values for the analyzed systems are usually very low, for, example,  $10^{-12}$ . In that case, at the bit-rate 155Mbps an error happens on average after each 6450s. At the bit-rate 2048 kbps the time interval between two successive errors is more than 500.000s. The measurement of BER corresponds to the Binomial distribution with the probability  $P_{BIN}$  according to the equation, [2]:

$$\begin{aligned} P_{BIN}(N_{ERR}, N_{BIT}, BER) &= \\ &= \frac{N_{BIT}!}{(N_{BIT} - N_{ERR})!} \cdot BER^{N_{ERR}} \cdot \\ &\cdot (1 - BER)^{N_{BIT} - N_{ERR}} \end{aligned} \quad (4)$$

The value  $P_{BIN}$  expresses the probability that some number of errors happens in the total number of transmitted bits  $N_{BIT}$  for BER, [2], [6].

In the case that BER value is relatively low ( $BER < 10^{-4}$ ), and the total number of received bits is high ( $N_{BIT} > 10^5$ ), Poisson distribution approaches Binomial distribution and it is possible to use simpler Poisson distribution. In order to express the probability of some number of erroneous bits by Poisson distribution, it is necessary to define the parameter  $\mu$ , which represents the probability of one erroneous bit transmission. The parameter  $\mu$  may be defined as, [2], [6]:

$$\mu = BER \cdot N_{BIT}. \quad (5)$$

In such a case the Probability Density Function (PDF) of Poisson distribution for the a priori defined value of BER as the measurement experiment:

$$P_{POISS}(N_{ERR}, \mu) = e^{-\mu} \cdot \frac{1}{N_{ERR}!} \cdot \mu^{N_{ERR}}, \quad (6)$$

where  $N_{ERR}$  must be an integer, while  $\mu$  may be any non-negative real number.

On the base of relations (4), (5), (6) and the BER values, it is possible to determine the probability of  $N_{ERR}$  erroneous bits in a total number of bits  $N_{BIT}$ . It is also possible to determine the total number of transmitted bits  $N_{BIT}$  for some value of BER with the desired accuracy.

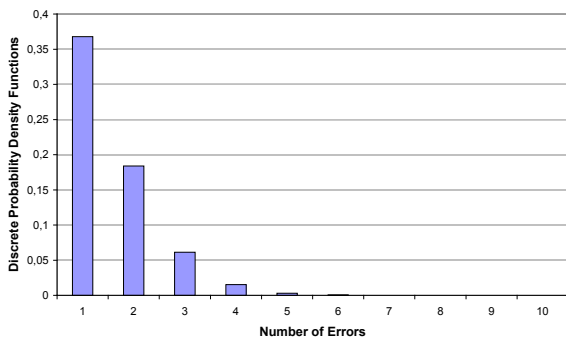
## THE MEASUREMENT ACCURACY ESTIMATION OF ERRONEOUS BITS RELATIONS

The measurements realized on the base of PDF which defines BER allow presentation of accuracy estimations. As an example, it is possible to compare three BER values on a system with real  $BER = 10^{-12}$  and the only compared number of bits:

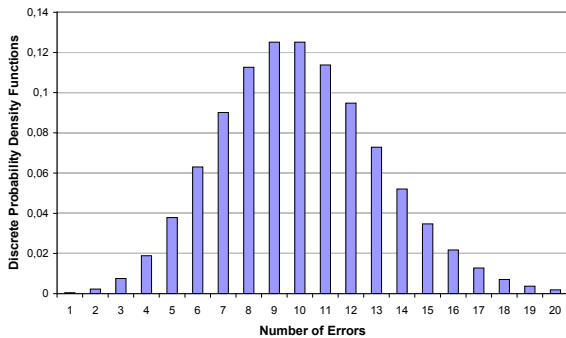
- $N_{BIT} = 10^{12}$  ( $\mu = 1$ ). The probability to obtain one error in a test (which corresponds to the exact  $BER = 10^{-12}$ ) is 0.3679;

- $N_{BIT} = 10^{13}$  ( $\mu = 10$ ). The probability of 10 errors (BER =  $10^{-12}$ ) is only 0.1215;
- $N_{BIT} = 10^{14}$  ( $\mu = 100$ ). The probability of 100 errors (BER =  $10^{-12}$ ) is even lower. i.e. 0.0399.

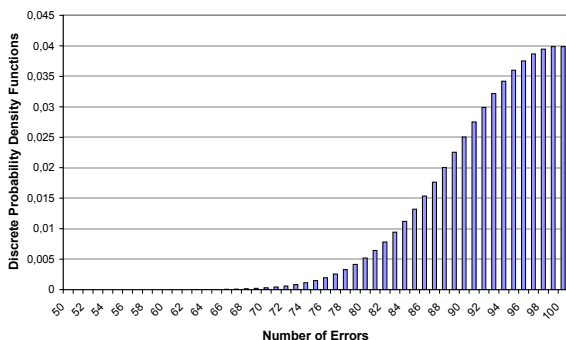
Does it mean that the results will be better if the lower number of bits **is compared**? The situation is quite the opposite. The Figure 1 presents discrete PDF values for various  $\mu=1$ ,  $\mu=10$  and  $\mu=100$ . The absolute probability values are really higher for  $\mu=1$ , but only because there is lower number of possible outcomes.



a)



b)



c)

**Fig. 1.** Probability Density Functions for a Poisson distribution with  $\mu=1$  a),  $\mu=10$  b) and  $\mu=100$  c)

It is interesting to notice that for  $\mu=1$  the probability of no errors (BER=0) is completely the same as the probability of one error

(BER= $10^{-12}$ ). But, for  $\mu=10$  the probability that there is no error is nearly zero ( $4.54 \cdot 10^{-5}$ ). Besides, the probability of two errors for  $\mu=1$  (BER =  $2 \cdot 10^{-12}$ , two-fold real value) is 0.1839, but the probability of 20 errors in a case  $\mu=10$  (for the same BER) is only 0.00187. If **it is supposed** that only one incorrectly transmitted bit  $N_{ERR}=1$  **is considered** and that the accuracy in error determination  $P_{POISS}(N_{ERR}, \mu) = 0.99$  **is expected**, minimum and maximum values of parameter  $\mu$  may be determined numerically. For the known BER value according to (5) the number of transmitted bits  $N_{BIT}$  **may be also determined**. The values  $\mu_{min} = 0.1486$  and  $\mu_{max} = 6.6384$  for  $N_{ERR} = 1$  and  $P_{POISS}(N_{ERR}, \mu) = 0.99$  are determined using the program Microsoft Excel. Min and max total number of received bits,  $N_{BIT}$ , for the specified BER are calculated using (5) and presented in Table 1.

Starting from the total number of bits,  $N_{BIT}$  and the transmission rate,  $v_i$  in bps, the necessary measurement time **may be determined** as,

$$t_{mer} = \frac{N_{BIT}}{v_i} \quad (7)$$

The calculated minimum values of measurement time to determine the value of BER are presented in Table 1.

**Table 1.** Measurement time for the transmission speed 2.048 Mbps in the range of  $10^{-6} - 10^{-14}$

BER	$N_{BIT}$ (bit)		$t_{mer}$	
	Min	Max	Min	Max
$10^{-14}$	$1.49 \cdot 10^{13}$	$6.64 \cdot 10^{14}$	84d 06:13:20	3750d
$10^{-13}$	$1.49 \cdot 10^{12}$	$6.64 \cdot 10^{13}$	8d 10:13:20	375d
$10^{-12}$	$1.49 \cdot 10^{11}$	$6.64 \cdot 10^{12}$	0d 20:13:20	37d 12:00:00
$10^{-11}$	$1.49 \cdot 10^{10}$	$6.64 \cdot 10^{11}$	0d 02:01:20	3d 18:00:00
$10^{-10}$	$1.49 \cdot 10^9$	$6.64 \cdot 10^{10}$	0d 00:12:08	0d 09:00:00
$10^{-9}$	$1.49 \cdot 10^8$	$6.64 \cdot 10^9$	0d 00:01:13	0d 00:54:00
$10^{-8}$	$1.49 \cdot 10^7$	$6.64 \cdot 10^8$	0d 00:00:07	0d 00:05:24
$10^{-7}$	$1.49 \cdot 10^6$	$6.64 \cdot 10^7$	0d 00:00:01	0d 00:00:32
$10^{-6}$	$1.49 \cdot 10^5$	$6.64 \cdot 10^6$	0d 00:00:00	0d 00:00:03

## QUALITY FACTOR

Quality factor (Q-factor) determines the quality of digital signal from the analog viewpoint. That's why it is supposed to be the relation signal/noise. In practical measurements it is possible to determine the difference in signal level comparing to the noise level and additionally to estimate parameters as BER and Q-factor [7], [8], [9].

The difference between the signal level and noise level may be determined from the equation:

$$OSNR = 10 \log\left(\frac{P_i}{N_i}\right) + 10 \log\left(\frac{B_m}{B_r}\right), \quad (8)$$

where it is:

$P_i$  – mean power of optical signal in the  $i^{\text{th}}$  channel [W];

$N_i$  – interpolated value of the mean noise power [W];

$B_m$  – spectral bandwidth where measurements are performed [nm];

$B_r$  – reference bandwidth.

Quality factor may be calculated from the following equation (Figure 2):

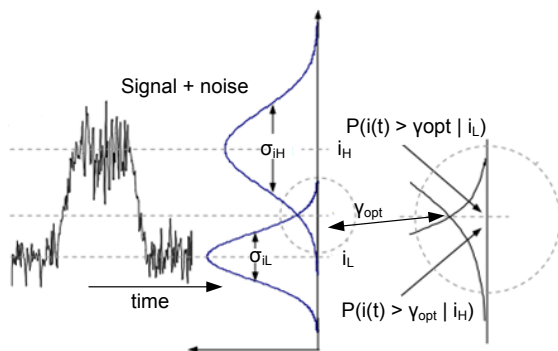
$$Q = \frac{i_H - \gamma_{opt}}{\sigma_{iH}} = \frac{\gamma_{opt} - i_L}{\sigma_{iL}}, \quad (9)$$

where it is:

$\gamma_{opt}$  – optimum decision threshold,

$i_L$  – the current which corresponds to the optical power level on a foto detector for the logical 0 signal,

$i_H$  – the current which corresponds to the optical power level on a foto detector for the logical 1 signal.



**Fig. 2.** Probability of error for binary signalling, [11]

If  $\gamma_{opt}$  is eliminated from (9), it is obtained the relation:

$$Q = \frac{i_H - i_L}{\sigma_{iH} + \sigma_{iL}} \quad (10)$$

## THE RELATION BETWEEN BIT ERROR RATE AND QUALITY FACTOR

Quality factor expresses the quality of DWDM systems, [2], [11], [12]. The characteristic of these systems is reliable transmission of high data quantities with small error probability. According to this consideration, error measurement in classical way would be very long-term, as presented in Table 2.

Synchronous Transport Module, STM is designated abbreviated as STM-x, where x is the variable representing the multiplier of the basic rate 155.52. Therefore, the transmission rate of STM-1 system is 155.52 Mbit/s, while the transmission rate of STM-4 is 622.08 Mbit/s ( $4 \cdot 155.52$  Mbit/s).

The transmission rate of Optical Carrier (OC) presents transmission speed of optical network and it is designated as OCK, where k is variable representing the multiplier of the basic rate 51.84 Mbit/s. Therefore, OC-1 fibers have the transmission rate 51.84 Mbit/s, while the transmission rate of OC-3 network is 155.52 Mbit/s ( $3 \cdot 51.84$  Mbit/s).

**Table 2.** Relationship between the length measurement of BER and Q-factor

BER	$10^{-12}$	$10^{-13}$	$10^{-14}$	$10^{-15}$	$10^{-16}$	Speeds
STM-16/OC-48	7 min	70 min	11 hrs	6 days	46 days	2.5 Gb
STM-64/OC-192	2 min	17 min	3 hrs	28 hrs	12 days	10 Gb
STM-69/OC-420	1.75 min	2.05 min	1.83 hrs	2.57 hrs	1.9 days	20 Gb
STM-256/OC-768	52.5 sec	1.03 min	54.9 min	1.29 hrs	57 hrs	40 Gb
STM-640/OC-1920	12 sec	1.7 min	18 min	2.8 hrs	1.2 days	100 Gb
STM-1234/OC-3840	10.5 sec	12.3 sec	10.98 min	15.42 min	4.96 hrs	200 Gb

If suppose that the probabilities of bit values 0 and 1 are equal, the relation between BER and Q factor may be expressed as [13]:

$$\begin{aligned}
 BER &= P(Error) = \\
 &= \frac{1}{\sqrt{2\pi\sigma_{iL}^2}} \int_{\gamma_{opt}}^{\infty} e^{-\frac{1}{2}\left(\frac{i-i_L}{\sigma_{iL}}\right)^2} di = \quad (11) \\
 &= \frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{1}{Q \cdot \sqrt{2} \cdot \pi} \exp\left(-\frac{Q^2}{2}\right)
 \end{aligned}$$

If the equation (11) is used for some range of Q-factor values, it is obtained the relation to BER, as presented in Figure 3. It is obvious from the Figure 3 that for the value  $Q = 6$  BER is  $1.012647 \cdot 10^{-9}$ , while for  $Q = 7$  BER is  $1.304960 \cdot 10^{-12}$ .

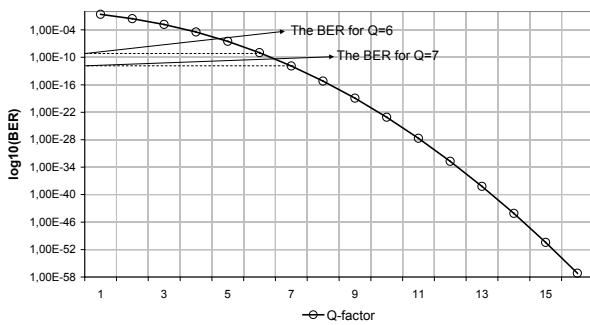


Fig. 1. The relation between BER and Q-factor

## CONCLUSION

Optical fibers are dominant in basic networks and the development is realized by DWDM multiplexing of wavelengths. The purpose of optical hierarchy is to compose the common platform for different network types to support the advanced service parts. In this paper the goal was to explain the connection between error parameters in optical wireless networks and nodes, leading to the conversion of optical signals into electrical signal [2], [10]. Further, the relation between Q-factor and BER is analyzed. The rate measurement, the independent digital signal structure and the wide area of transmission rates nominate Q-factor for DWDM systems monitoring.

## ACKNOWLEDGEMENT

This paper is realized in the framework of project TR 32007 and TR32051, which is cofinanced by Ministry of Education, Science and Technological Development of Republic of Serbia.

## REFERENCE

- [1] Stamatios V. Kartalopoulos. Introduction to DWDM Technology. Chapter 14: DWDM Systems, IEEE Communications Society. Sponsor. New York: John Wiley and Sons, Inc. Publication, 2000.
- [2] Tomáš Ivaniga, Petr Ivaniga. Evaluation of the bit error rate and Q-factor in optical networks. IOSR Journal of Electronics and Communication Engineering (IOSR-JECE). Nov - Dec. 2014; 9(6): 01-03.
- [3] Andrea Goldsmith. Wireless Communications. by Cambridge University Press, 2005.
- [4] Jahangir Alam S M, Rabiul Alam M, Guoqing Hu, Zakirul Mehrab Md. Bit Error Rate Optimization in Fiber Optic Communications. International Journal of Machine Learning and Computing, December 2011: 1(5).
- [5] Ramin Khalili, Kavé Salamatian. A new analytic approach to evaluation of packet error rate in wireless networks. 3rd Annual Communication Networks and Services Research Conference (CNSR'05). 2005. DOI:10.1109/CNSR.2005.14.
- [6] Müller M, Stephens R, Mchugh R. Total Jitter Measurement at Low Probability Levels. Using Optimized BERT Scan Method. February 12. 2007. [www.agilent.com/find/dca, http://literature.cdn.keysight.com/litweb/pdf/5989-2933EN.pdf](http://literature.cdn.keysight.com/litweb/pdf/5989-2933EN.pdf)
- [7] Neha Vaishampayan. Comparative Analysis of Bit Error Rate and Quality Factor at Different Power Levels over Fiber Optic Link. IJSD - International Journal for Scientific Research and Development. 2015; 3(8).
- [8] Anis AA, Rashidi CBM, Rahman AK, Aljunid SA, Ali N. Analysis of the effect of BER and Q-factor on free space optical communication system using diverse wavelength technique. EPJ Web of Conferences 162 01024, 2017. DOI: 10.1051/epjconf/201716201024.
- [9] Lian K Chen, Calvin C K Chan, Lu GW, Ku YC, Ho ST, Chinlon Lin. Optical Performance Monitoring and Network Diagnosis in Reconfigurable Optical Networks. Proceedings of SPIE - The International Society for Optical Engineering, December 2007. DOI: 10.1117/12.747236.
- [10] Maxim Integrated. Optical Signal-to-Noise Ratio and the Q-Factor in Fiber-Optic Communication Systems. Application Note HFAN-9.0.2. Rev.1. 04/2008.
- [11] Tony Antony, Ashwin Gumaste. WDM Network Design. Chapter 4: Design of a Point-to-Point Link Based on Q-Factor and OSNR. Chapter 5: Calculation of Q-Factor from OSNR

Published Feb 7. 2003 by Cisco Press. Part of the Networking Technology series.

- [12] Ashwin Gumaste, Tony Antony. DWDM Network Designs and Engineering Solutions. Chapter 2: Design and evaluate optical components in a DWDM network. Chapter 6: Design optical links based on OSNR. Chapter 7: Design a real DWDM network with

impairment due to OSNR, dispersion and gain tilt. Part of the Networking Technology series. Published Dec 13. 2003 by Cisco Press.

- [13] ITU-T O.201. Series O: Specifications of Measuring Equipment, Q-factor test equipment to estimate the transmission performance of optical channels. 07/2003.