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ERROR ESTIMATION FOR NETWORKS WITH HIGHER DATA RATES

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Abstract

The decisive factor for determination of the quality in higher speed digital networks is Bit Error Rate (BER). BER definition in the Recommendation G.821 is based on measuring the number of errored bits, which significantly complicates practical measurements. The Recommendation G.826 defines qualitative parameters and end points for BER estimation in higher speed networks and overcomes the problems whose answer is not specified in the Recommendation G.821. The variable BER for data transmission in optical networks is defined in the Recommendation G.8201, which follows Recommendations G.826, G.828 and G.829. Asymptotic method for error performance parameters prediction based on normal approximation of binomial distribution is going to be defined. A simple conversion method, which is developed for the case of time variant with the constant index of BER(t) distribution and invariant mean threshold length is also going to be presented.

Keywords: Bit Error Rate, Errored second , Errored block , Severely errored second, probabilities.

INTRODUCTION

The Recommendation ITU G.826 [1] defines qualitative parameters and end points for BER estimation for higher data flow-rate transmission networks. The Recommendation is intended for the hypothetical reference connection of 27500 km, for different transmission media such as optical fiber, classical lines with metallic conductor wire or satellite transmission systems. This Recommendation is supplemented by the more precise specification for synchronous, plesiochronous and asynchronous systems [2] for its practical implementation. The errors in the block [1], error units and parameters are defined in the Section 2. Asymptotic event probabilities P_{EB}, P_{ES} and P_{ESE} are defined in the Section 3, while the analysis of event probabilities P_{EB}, P_{ES} and P_{ESE} as a function of time is given in the Section 4. The error definition for modern SDH systems is presented in the Section 5, while error explanation for the Optical Transport Network, OTN [3] is the subject of the Section 6. At the end, the conclusions are emphasized in the Section 7.

DEFINITION OF ERRORS IN A BLOCK, ERROR UNITS AND PARAMETERS

While defining the Recommendation G.826, it was set the main goal to model the BER characteristics for the measurements in system function, thus enabling implementation of the equipment for error monitoring on the instantaneous transmission systems. However, the measuring technics had to be removed from the field of measuring errored bits, according to the Recommendation G.821 [3], onto the field of measuring errored blocks.

Monitoring of errors in block uses existing transmission systems based on Cyclic Redundancy Check (CRC 4) and bits Parity Control. According to the Recommendation G.826, the errors should be detected with the probability of at least 90%. This request is satisfied for CRC 4 and BIP-8, but not for BIP-2 (Bit Interleaved Parity, BIP) [4], [5], [6].

The Recommendation ITU G.826 defines four main parameters for BER estimation in a block:

- **Errored Block (EB):** a block in which one or more bits are in error;
- **Errored Second (ES):** A one second period with one or more errored blocks or at least one defect;
- **Severely Errored Second (SES):** a one-second period, which contains $\geq$ than 30% errored blocks or at least one defect. SES is a subset of ES.
- **Background Block Error (BBE):** an errored block not occurring as a part of SES.

Severely disturbed time domains measured in practice for Plesiochronous Digital Hierarchy (PDH) and Synchronous Digital Hierarchy (SDH) systems are defined differently. In this category several alarm types are defined, as: signal loss, frame synchronization loss, multiframe synchronization loss, alarm indication signal (AIS).

Severely disturbed time domains occur when the error rate is greater than 10^{-3}. Measurements of error variables give absolute values of these variables. However, it is more useful to use relative values. The recommendation G.826 defines three parameters of relative error [7]:

- **Errored Second Ratio (ESR):** the ratio of ES to total seconds in available time during a fixed measurement interval;
- **Severely Errored Seconds Ratio (SESR)**: the ratio of SES to total seconds in available time during a fixed measurement interval;
- **Background Block Error Ratio (BBER):** the ratio of errored blocks to total blocks during a fixed measurement interval, excluding all blocks during SES and unavailable time.

These error parameters are valid if the system is in operation. According to the Recommendation ITU G.826, a system is supposed to be defective if the BER value is at least 10 seconds higher than the allowed BER for the considered transmission system. The system becomes operative after at least 10 consecutive seconds which are not of SES type [1], [7].

P_{EB}, P_{ES} AND P_{SES} - ASYMPTOTIC PROBABILITIES OF THE EVENTS EB, ES AND SES

The relation between error performance parameters and BER may be estimated using Poisson distribution or Neiman-A bit error distribution during an error burst. As Poisson distribution is more pragmatic and offers useful results with minimum complexity, it is going to be implemented [8].

THE PROBABILITY OF ERRORED BLOCK P_{EB}

Assuming that the distribution of errored bursts is Poissonian, the probability of the errored block P_{EB} may be expressed as:

$$P_{EB} = 1 - \exp\left(-\frac{N_B \cdot BER}{\alpha}\right) = 1 - \exp\left(-\frac{BER}{\tau}\right), \qquad (1)$$

where N_B is the number of bits per block, α is the mean number of errors per burst ($\alpha \geq 1$), the constant $\tau = \alpha/N_B$ is used to estimate the rate of exponent variation. On the base of fact that the value of negative exponential function is changed from 1 to 0.05 when the value of exponent is changed from 0 to $3 \cdot \tau$, it is possible to be concluded that when BER is changed from 0 to $3 \cdot \alpha/N_B$, the probability of errored block P_{EB} is changed from 0 to 0.95.

This relation which makes connection between the area of P_{EB} values with the block size and the number of errors per burst is very useful when it is necessary to perform numerical integration.

THE PROBABILITY OF ERRORED SECOND P_{ES}

Starting from the probability of errored block according to the equation (1), it is possible to define the probability of the errored second P_{ES} as:

$$P_{ES} = 1 - \exp(-n \cdot P_{EB}), \qquad (2)$$

where n is the number of blocks per second. On the base of „rule 3·τ", it may be concluded:

P_{ES} = 0.95 if it is P_{EB} = 3/n, (3)

i.e. P_{ES} is in the interval $0 \le P_{ES} \le 0.95$ where P_{EB} is divided by 1/n. According to this statement, the linear approximation of the equation (1) may be formed and it is possible to write the expression:

$$P_{ES} = 1-\exp\left(-n\cdot\frac{N_B\cdot BER}{\alpha}\right) = \\ = 1-\exp\left(-n\cdot\frac{BER}{\tau}\right), \quad (4)$$

which is practically accurate (the error is less than $1/n^2$). Comparing the expressions (1) and (4), it is possible to conclude that the following relation is valid for any fixed value of BER and the same parameters:

$$P_{EB}(BER) = P_{ES}(BER/n). \quad (5)$$

It is obvious from the equation (5) that the agreement exists between different BER values, which assure the same probability of errored blocks and errored seconds, $P_{EB} = P_{ES}$. For example, from the Figure 1 BER=$1.91\cdot10^{-5}$ for P_{EB}=0.3 in the case of VC-4 container when it is N_B = 18720, n = 8000 and taking into account the values $P_{EB}=P_{ES}$=0.3. If this BER value is divided by n=8000, according to the equation (5), it is obtained (see Figure 1):

$$P_{EB}\left(1{,}91\cdot10^{-5}\right) = P_{ES}\left(2{,}3\cdot10^{-9}\right) = 3. \quad (6)$$

THE PROBABILITY OF SEVERE ERRORS IN SECOND P_{SES}

In order to estimate the probability of errored seconds, it is first necessary to define the probability P_{nk} where k is the number of errored blocks of total n blocks. It is realized using binomial distribution as:

$$P_{nk} = \left[\frac{n!}{(n-k)!}\right]\cdot\left[\left(1-P_{EB}^{n-k}\right)\cdot\left(P_{EB}\right)^k\right]. \quad (7)$$

Normal distribution with the main value μ and variance σ^2 is defined as [9]:

$$\mu = n\cdot P_{EB},\ \sigma^2 = n\cdot P_{EB}\cdot(1-P_{EB}), \quad (8)$$

and it gives very accurate approximation of binomial distribution when n is great. As a consequence it is possible to write the equation (7) in the case that n tends to infinity as:

$$\lim_{n\to\infty} P_{nk} = f(x) = \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot e^{\frac{-(x-\mu)^2}{2\cdot\sigma^2}}. \quad (9)$$

The random normal variable n in the expression (10) represents the number of errored blocks in second. Considering all cases when it is $(k/n) \ge 0.3$ the probability P_{SES} may be calculated as the sum:

$$P_{SES} = \sum_{k=0.3\cdot n}^{n} P_{nk}, \quad (10)$$

and using (9) this expression may be replaced by integral. It means that P_{SES} is then:

$$P_{SES} = \int_{x=0.3\cdot n}^{n} f(x)dx. \quad (11)$$

As earlier, f(x) is the probability density function of normal random variable x with mean value μ and variance σ^2. The characteristics of integral (11) are explained in detail in [10, Supplement A]. The main conclusion may be summarized as the statement that the probability P_{SES} may be expressed in normal distribution as:

$$P_{SES} = 1-F(u_1),\ u_1 = \frac{0.3\cdot n-\mu}{\sigma} = \\ = \frac{n(0.3-P_{EB})}{\sqrt{n\cdot P_{EB}\cdot(1-P_{EB})}}, \quad (12)$$

where $F(u_1)$ is cumulative standard normal distribution of random variable u with the mean value equal to zero and variance equal to 1, where u_1 is normalized lower limit of the integral (11).

The probability P_{SES} is changed in the interval [0.005, 0.995] if the probability of errored block P_{EB} is changed for 0.3±3. The corresponding BER is defined as:

$$0.005 \le P_{SES} \le 0.995 \text{ under} \\ BER = B_{SES} \pm 3\cdot(\alpha/N_B). \quad (13)$$

Therefore, the expression (13) proves in an analytical way the commonly accepted assumption that P_{SES} is really a step function. The graphs of P_{EB}, P_{ES} and P_{SES} as a function of BER are presented in Figure 1 for a VC-4 link.

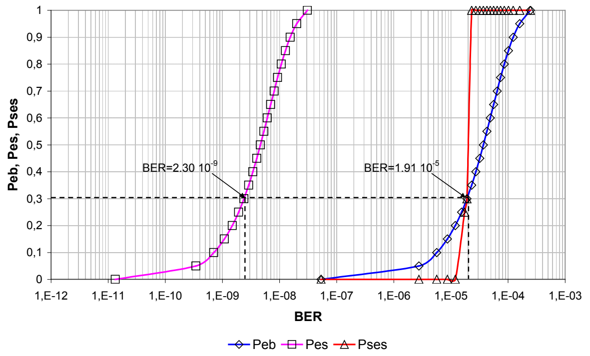

Fig. 1. *The presentation of P_{EB}, P_{ES} and P_{SES} as a function of BER for the VC-4 link.*

THE PROBABILITY OF EVENTS P_{EB}, P_{ES} AND P_{ESE} AS A FUNCTION OF TIME

Under the remainder of the function BER (t), in the case of the Danish domain, the data shall be [10]:

$$BER(t) = B_0 \cdot \left(\frac{t_0}{t}\right)^m, \tag{14}$$

where m is a known value greater than one.

When the relation for BER(t) is known, the error probabilities may be expressed as a function of relative time t. If Poissonian distribution of error cluster is supposed, the probability of errored block P_{EB} is written as [11], [12]:

$$P_{EB} = 1 - \exp\left(-N_B \cdot \frac{BER}{\alpha}\right), \tag{15}$$

where it is N_B – number of bits in a block and α – the mean number of errors in a cluster (α ≥ 1).

If BER(t) in the equation (15) is replaced, the new expression for P_{EB} as a function of time is obtained as

$$P_{EB} = 1 - \exp\left(-N_B \cdot \frac{B_0}{\alpha} \cdot \left(\frac{t_0}{t}\right)^m\right). \tag{16}$$

The probability that there is an error in a second, P_{ES}, is presented as:

$$P_{ES} = 1 - \exp(-n \cdot P_{EB}), \tag{17}$$

where it is n – number of blocks in a second, and P_{EB} – probability that there is an errored block.

If the expression for P_{EB}, equation (16) is replaced in the equation (17), the new expression for P_{ES} as a function of time is:

$$P_{ES} = 1 - \exp\left(-N_B \cdot n \cdot \frac{B_0}{\alpha}\left(\frac{t_0}{t}\right)^m\right). \tag{18}$$

Now it is possible to calculate the probability of Severely Errored Second (SES) P_{SES} as a second interval with at least 30% errored blocks [1], [13].

$$P_{SES} = 1 - F[u_1(t)], \tag{19}$$

where $F[u_1(t)]$ is a function of normal distribution and $u_1(t)$ is presented as:

$$u_1(t) = \frac{0.3 \cdot n - n \cdot P_{EB}(t)}{\sqrt{n \cdot P_{EB}(t) \cdot (1 - P_{EB}(t))}}. \tag{20}$$

The probabilities $P_{EB}(t)$, $P_{ES}(t)$ and $P_{SES}(t)$ for a given time dependence may be used for an integration of relevant functions in relative time interval [0,1]. The parameters ESR and SESR may be calculated as:

$$\text{ESR} = \int_{t_0}^{1} P_{ES}(t)dt, \tag{21}$$

$$\text{SESR} = \int_{t_0}^{1} P_{SES}(t)dt, \tag{22}$$

where the lower limits, t_0, are the part of time when the error is greater than its highest value 10^{-3}. The upper limit, which is equal to one, corresponds to the considered time [1], [13].

Based on the definition from ITU-T G.826, BBER may be estimated using integral:

$$\text{BBER} = \int_{t_{SES}}^{1} P_{EB}(t)dt, \tag{23}$$

where the lower limit of integration is taken as a part of time, which corresponds to the Severely Errored Second (SES), with the value $P_{EB} = 0.3$. For $t > t_{SES}$ (SESR = $t > t_{SES}$) there is not a characteristic of severely errored second.

The equation (23) is useful from two reasons. The first one is that it gives implicit, in very simple way, the relation between BBER and P_{EB}, and the second one is that it significantly simplifies numerical integration [7], [10]. Trigonometric, the integrals from equations (21), (22) and (23) correspond to the graphs in Figure 2.

Figure 2 presents the curves for $P_{EB}(t)$, (16), $P_{ES}(t)$, (18) and $P_{SES}(t)$, (19) with the following parameters: N_B = 18792; B_0 = 0.001; α = 1; t_0 = 0.0001; m = 10; n = 8514 and n = 17028.

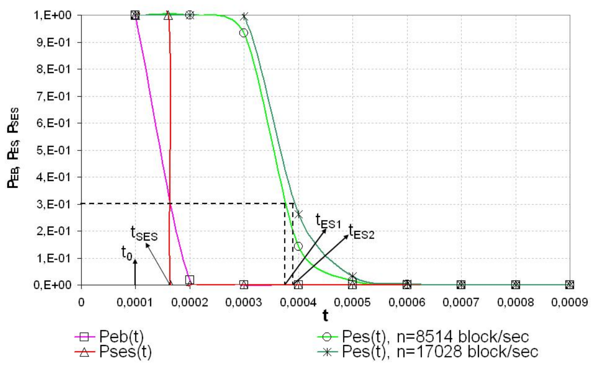

***Fig. 2.** Presentation of the curves $P_{EB}(t)$, $P_{ES}(t)$, and $P_{SES}(t)$ for VC-4 link*

It is obvious from the Figure 2 that P_{SES} is the step function. That's why the simple relation may be used to calculate SESR with the significant precision:

$$SESR = t_{SES} = 1.486518 \cdot 10^{-4} \,. \qquad (24)$$

It is also evident that the value t_{ES2} = $3.938097 \cdot 10^{-4}$, the graph for n = 17028, is higher than the value $t_{ES1} = 3.674374 \cdot 10^{-4}$, the graph for n = 8514, all other parameters are the same.

THE ERROR DEFINITION FOR THE NEWER SDH SYSTEMS [14]

The target values which are defined in the Recommendation G.826 do not correspond to the modern SDH systems, where the physical layer transmission is realized by the optical fiber technology. The allowed error rates for modern SDH systems according to G.828 are stricter than according to G.826. The Recommendation G.828 is pretty the same as G.826. The great emphasis is on the possibility to perform measurements during the system functioning. The Recommendation defines the new error type Severely Errored Period (SEP), which is based on the results of practical measurements. SEP is defined as the time period with more than 3 and less than 9 consecutive seconds with severe errors. SEP may have the same effect as the micro-interruption and may lead to the high impairment of Quality of Services which are supported by SDH. The parameter which corresponds to the period with severe errors is called Severely Errored Period Intensity (SEPI) and its unit is 1/time. The suggested value of this parameter is 518 micro-pauses per month. The new Recommendation takes into account tandem connections monitoring, which is especially useful for modern SDH systems.

ERRORS FOR THE OPTICAL TRANSPORT NETWORKS, OTN [3]

The new Recommendation G.8201 defines the error rate characteristics for transmission in optical networks. The structure of G.8201 is very similar to the G.82x recommendation. It is also based on the principle of errored block measurement by code errors detection, which is very useful in operational measurements. The new term, so called domain operator, is established in the hypothetical reference optical trace. The border between domains is called Operator Gateway (OG). In order to keep the continuity with the Recommendations G.826 and G.828, the local and regional operator domain is related to the national part and to the main operator domain with the international part. To keep the further consistency with G.826 and G.828, four main operator domains are used (one for each transit country) and two pairs of local and regional operator domains, total 8 operator domains. The allocation principles may be implemented to achieve the desired error rate for national and private optical links. Hypothetical reference optical link is presented in the Figure 3 [1], [14], [15], [16],

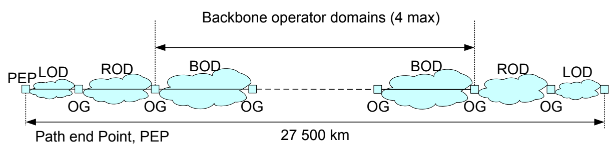

***Fig. 3**. Hypothetical reference optical link according to G.8201, [16]*

CONCLUSION

The instantaneous number of errored bits in an errored block is not important for the estimation according to the Recommendation G.826, except if it exists the period with severe errors. The Recommendation G.826 tolerates errored bits appearing in clusters, because in this case the high number of errored bits in one second causes lower number of errored blocks. In this paper the asymptotic method for error performance parameters prediction based on normal approximation of binomial distribution has been presented. It is determined the simple relation between the thresholds for P_{EB} and P_{ES}. It is proved that P_{SES} (Figure 2) s practically the step function and that it completely depends on the mean BER value and the block dimension.

The simple conversion method is developed for the case of time variant with the constant distribution index BER(t) and invariant mean threshold length. The analytical relations between ESR, SE and EB, which depend on the number of blocks in a second and BER distribution index, are determined under these conditions.

It is also elaborated the prediction method and the estimation of gradual treatment for the complete distribution BER(t) with the mean time variant of errors number.

It is also possible to notice in the Figure 1 that P_{SES} is step function and that it changes its value from 0 to 1 in a short time interval.

The new Recommendation G.828 takes into account tandem connection monitoring. It also defines new parameters SEP and SEPI, which tighten the criteria for error characteristics, thus making an effect to the ESR and BBER parameter. The Recommendation G.8201 defines error parameters and characteristics for optical transmission systems. Due to higher transmission rate ESR loses its meaning in the Recommendation G.8201, and only the parameters SESR and BBER are defined. The Recommendation G.8201 is still in the phase of redefinition.

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