APPLICATION OF THE PHASE LOCKED LOOP FOR THE DIGITAL FILTERING OF THE PULSE SIGNALS PERIODS

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Abstract

This work describes a new type of a Time Recursive Processing Phase Locked Loop (TRP PLL) which possesses three outputs. TRP PLL is described by difference equations. Analyses of the circuit in the time domain is made using the Z transform approach. It was shown that the circuit can be used for the digital filtering of the pulse signals periods and the time intervals belonging to the periods. Computer simulations of TRP PLL functioning was made in the time domain to demonstrate its ability for noise suppression. However, using mat-lab tools, dedicated to design of digital filters, large part of article was devoted to the frequency analysis of TRP PLL and to the discovering of its filter properties.

Keywords: Digital circuits, PLL, FLL, Pulse circuits, Digital filtering.

INTRODUCTION

The time recursive processing PLL (TRP PLL) described in this paper calculates and generates an output period by using measurement and processing of the input and output periods, as well as the time difference between them. The terms "non-recursive" and "recursive" are closely related to digital filter theory. In fact, the finite impulse response (FIR) digital filters use the non-recursive processing because they only process the samples of an input signal. However, the infinite impulse response (IIR) digital filters perform recursive processing because they use both, the input and output signal samples.

The borrowing concept from digital filter theory is no accident. On the contrary, it is of multiple importance. But one of the main aim is the intention to utilize the powerful mat-lab tools and the theory of digital filters for the development and analyzes of TRP PLLs. Providing this is possible, it would not only be a great advance for theory of TRP PLLs, but it would also be an outstanding contribution to digital filter theory. This would mean that we could develop a new type of digital filter, designed specifically for impulse signals, that has many new features compared to the existing approaches to digital filters. The analysis of the features of the new approach, as well as the differences and advantages, require much more space. But let's just mention one of them. It is the fact that such a digital filter would not use A/D and D/A converters. Accordingly, the accuracy and simplicity of measuring filter data would be increased significantly, because in that case, time periods would be measured instead of the amplitudes of signals. This new approach in digital filtering would provide many facilities and improvements.

The theory and techniques for the developing of TRP PLL are basically very similar to the demonstrated ones through refs. [1-11]. The applicability of this approach is very wide. Period averaging circuit is described in [1] Frequency multiplier is described in [2]. Time shifters are described in [3, 4] and time/phase shifting in [5]. PLL and FLL for noise rejection are described in [6-8]. A wide range of the tracking and prediction applications are described in [6, 7, 9]. Most of the algorithms described in [1-11] are suitable for usage in a software form. Such a software predictor is described in [10]. The frequency analyzes of FLL in [11] are performed using mat-lab tools. The articles and books in [12-18] are used as theoretical base, for electronics implementation and for the development necessities.

DESCRIPTION OF CIRCUIT IN THE TIME DOMAIN

Let us suppose that the observation of the input/output processing starts at discrete time t_k. One general case of the time relation between an input signal Sin and an output signal Sop of the circuit, is shown in Fig. 1. The periods TI_0, TI_1,….TI_k, TI_{k+1}, and TO_0, TO_1,….TO_k, TO_{k+1}, as well as the time differences τ_0, τ_1, τ_2…..τ_k, τ_{k+1}, occur at discrete times respectively t_0, t_1, t_2,…..t_k, t_{k+1}. The discrete times t_0, t_1, t_2,….t_k, t_{k+1} are defined by the falling edges of the pulses of Sop in Fig. 1. Note that all variables, are distributed in time. This fact provides specific situation that time variables depend on time. Another way of consideration is that time variables depend on number of steps "k", where "k" is the number of the processed periods. In the following analyzes this another

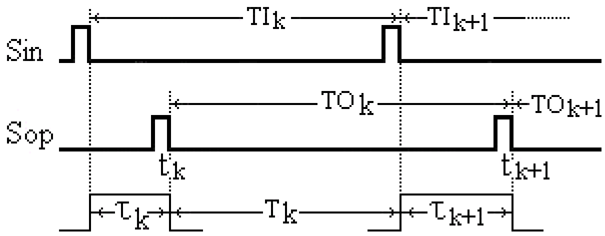

Fig.1. *Time relations between all variables*

way is adopted as more suitable. The main algorithm of the circuit is presented by (1), where "a" and "m" are the system parameters. The natural recursive relations (2) and (3), between the variables, yields from Fig. 1. It was assumed that time difference τ is positive if the output signal is delayed in phase. Accordingly, in case that the input signal is delayed in the phase, time difference τ will take the negative value through analysis. According to (1), (2) and (3), the circuit offers three available output variables, which describe the circuit behavior in the function of TI_k. The output variables are $TO(k+1)$=f[$TI(k)$], $\tau(k+1)$=f[$TI(k)$] and $T(k)$=f[$TI(k)$]. All of three variables can be easily and precise measured as time interval. It will be shown later on, that each of them possesses different filtering properties and that all of them can be useful for different applications. Note that, because of simplicity, $TO(k)$, $TI(k)$, $\tau(k)$ and $T(k)$ are often denoted in the article as TO_k, TI_k, τ_k and T_k. It is now necessary to find out the Z transform of the output variables $TO(k+1)$, $\tau(k+1)$ and $T(k)$ in order to analyze the properties of the circuit described. The Z transform of (1), (2) and (3) are given by respectively (4), (5) and (6). Time constants TO_0 and τ_0 are the initial values of the output variables $TO(k)$ and $\tau(k)$, which appear at discrete time $t_0 = 0$, i.e. at $t = t_k$ for $k = 0$. Calculating $\tau(z)$ from (5) and changing it into (4), $TO(z)$ was found out and shown in (7). Changing $TO(z)$ from (7) to (5), $\tau(z)$ was calculated and shown in (8). At last, changing $\tau(z)$ from (8) into (6), $T(z)$ was calculated and shown in (9).

$$TO_{k+1} = \mathrm{a} \cdot TO_k + m \cdot \tau_{k+1} \quad (1)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \quad (2)$$

$$T_k = TI_k - \tau_k \quad (3)$$

$$zTO(z) - zTO_0 = \mathrm{a} \cdot TO(z) + m \cdot \left[z\tau(z) - z\tau_0 \right] \quad (4)$$

$$z \cdot \tau(z) - z \cdot \tau_0 = \tau(z) + TO(z) - TI(z) \quad (5)$$

$$T(z) = TI(z) - \tau(z) \quad (6)$$

$$TO(\mathrm{z}) = \frac{-TI(\mathrm{z})mz}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} + \frac{\tau_0 mz + TO_0\,\mathrm{z}(\mathrm{z}-1)}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} \quad (7)$$

$$\tau(z) = \frac{-TI(z)(z-\mathrm{a})}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} + \frac{\tau_0 z(\mathrm{z}-\mathrm{a}-m) + TO_0 z}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} \quad (8)$$

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Three transfer functions, presented by (10), (11) and (12), describing the output variables of the circuit in dependence on the input period, can be defined from respectively (7), (8) and (9).

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{-\mathrm{m}\,z}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} \quad (10)$$

$$H_{\tau}(z) = \frac{\tau(z)}{TI(z)} = \frac{-(z-\mathrm{a})}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} \quad (11)$$

$$H_{T}(z) = \frac{T(z)}{TI(z)} = \frac{z^2 - z(\mathrm{a}+m)}{\mathrm{z}^2 - z(\mathrm{a}+\mathrm{m}+1)+\mathrm{a}} \quad (12)$$

Step analysis will discover under which conditions the circuit can possess the properties of a PLL. Let us suppose that the step function, *TI(k)=TI*=const., is applied to the input. If we change the Z transforms *TI(z)* = *TI·z/(z-1)* into eq. (7) and using the final value theorem, it is possible to find the final value of the output period in the time domain as TO_∞ = lim *TO(k)* if $k\to\infty$, using *TO(z)*, as it is shown in (13). To complete the information about the system properties, it is necessary to determine τ_∞, i.e. the final value of *τ(k)*. If *TI(z)* is changed into eq. (8), using the final value theorem in the same way like for TO_∞, τ_∞ was determined and shown in (14). In the same way T_∞ is calculated and shown in (15).

$$TO_\infty = \lim[(z-1)\cdot TO(z)]_{z\to 1} = TI \tag{13}$$

$$\tau_\infty = \lim[(z-1)\cdot \tau(z)]_{z\to 1} = TI(1-\mathrm{a})/\mathrm{m} \tag{14}$$

$$T_\infty = \lim[(z-1)\cdot T(z)]_{z\to 1} = -TI(1-\mathrm{a}-\mathrm{m})/\mathrm{m} \tag{15}$$

The expressions (13), (14) and (15) are valued only if the circuit is the stable system i.e. if $|z_1| < 1$ and $|z_2| < 1$, where z_1 and z_2 are the poles of the transfer functions, given by either (9) or (10) or (11). The poles are the zeros of polynomial z^2-z(a+m+1)+a. Since $z_{1/2} = (\mathrm{a}+\mathrm{m}+1)/2 \pm \sqrt{[(\mathrm{a}+m+1)/2]^2 - \mathrm{a}}$, it yields that the circuit is the stable system if eqs. (16) are satisfied. The field of the parameters for the stable system is shown in Fig. 2.

$$-2(\mathrm{a}+1) < m < 0 \quad \text{and} \quad 0 < \mathrm{a} < 1 \tag{16}$$

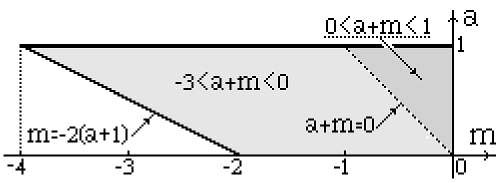

Fig.2. *The field of the circuit parameters*

Let us now simulate the behaviour of *TO(k)* and *τ(k)*, providing that step input is entered into TRP PLL. All discrete values in simulations were merged to form continuous curves. Note that all variables in the following diagrams were presented in time units. The time unit can be, μsec, msec or any other, but assuming the same time units for *TI*, *TO*, *T*, and *τ.* It was more suitable to use just "time unit" or abbreviated "t.u." in the text. It was more convenient to omit the indication „t.u." in diagrams. All simulations were performed using eqs. (1), (2) and (3) in time domain. The simulations of *TO(k)* and *τ(k)* for the step input TI_k=10 t.u, are shown in Fig. 3. There are three cases of simulations in Fig. 3 for a = constant = 0.1 and different values of parameter "m". All values for parameters, initial conditions and final values are shown in Fig. 3. The system parameters satisfy eq. (16) for the stable system in the cases Nr. 1 and Nr. 2. In these cases the output periods reached the input periods. However, in case Nr. 3, TRP PLL is not stable system and $TO_3\to\infty$ in Fig. 3. According to eq. (14), using the values of parameters presented in Fig. 3, it can be calculated $\tau_{1\infty}$ = TI(1-a)/m_1 = 10(1-0.1)/(-1) = -9 t.u and $\tau_{2\infty}$ = TI(1-a)/m_2 = 10(1-0.1)/(-0.5) = -19 t.u. Note that the calculated values $\tau_{1\infty}$, and $\tau_{2\infty}$ agree with the simulated $\tau_{1\infty}$, and $\tau_{2\infty}$ presented in Fig. 3. As it is expected, since TRP PLL is not stable system for m=-2.25 in case Nr. 3, simulated $\tau_{3\infty}$ in Fig. 3 tends to infinity. These simulation results prove the correctness of the mathematical description and step analyses. The real time relation between Sin, Sop and τ_k, for the simulated case Nr. 1, is shown in Fig. 4. Note that negative time difference τ are shaded in Fig. 4.

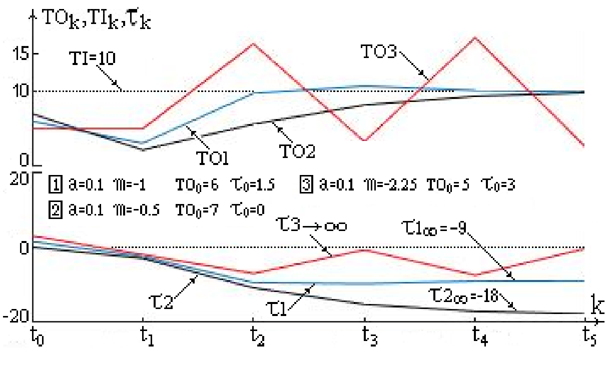

Fig.3. *Transition states of PLL for the step input, a=0.1 and different values of parameter m.*

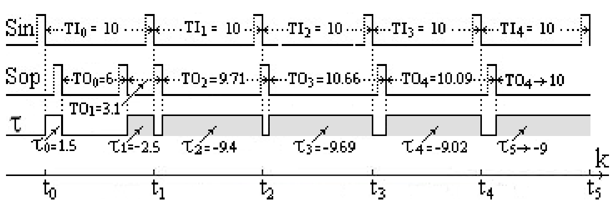

Fig.4. *Real time presentation of Sin, Sop and τ_k for the simulated case Nr 1, shown in Fig. 3.*

DIGITAL FILTERING OF PERIODS AND TME INTERVALS

It was proved in ref. [11] that mat-lab tools, devoted to the design of digital filters, can be used to analyze the TRP FLLs and TRP PLLs in the frequency domain. The main purpose of this chapter is to demonstrate the various filter characteristics of the TRP PLL described and to show that TRP PLLs are, at the same time, a kind of digital filters, which possess power ability to filter the pulse signal periods, as well as the time intervals, belonging to the periods. To demonstrate this, let us consider the filter characteristics of TRP FLL for the constant system parameter "a" and for the different parameters "m". According to Fig. 2, there is a very great possibilities to adapt the filter characteristics of TRP PLL to a particular application. To illustrate this, mat-lab command "freqz" and the transfer function $H_{TO}(z)$, given by eq. (10), are used. Entering the corresponding vectors of $H_{TO}(z)$ coefficients into command "freqz", the magnitudes of frequency responses of TO_k in the regain (0, pi) [rad], for a=constant=0.5 and for three values of "m" (m_1=-0.5, m_2=-1.5 and m_3=-2.6), are presented in Fig. 5. Note that for m=-0.5 TRP PLL behaves as a low-pass filter, for m=-1.5 TRP PLL behaves as a band-pass filter and for m=-2.6 TRP PLL behaves as a high-pass filter. All of three filters provide suppression of 0 [dB] at 0 Hz. This means that

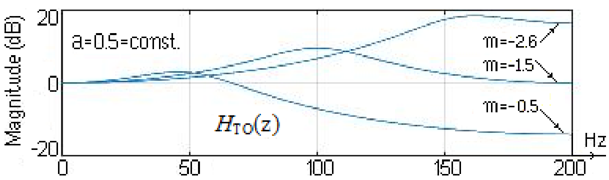

Fig.5. *Magnitude of frequency response of $H_{TO}(z)$ for a=0.5 and different "m"*

TRP PLL, providing that any pulse rate TI_k with constant input period is entered, would generate the same constant period at the output TO_k. This feature is consistent with any other PLL. This property of TRP PLL described, has been already proved by eq. (13). Changing of parameter "a" would provide additional adapting of the filter properties to the application requirements. It can be seen that the filter edges in Fig. 5 are not sharp. But this is expected, since TRP PLL is the system of the second order. Using TRP PLL of a higher order and choosing the corresponding system parameters, it would be possible to generate sharper band edges, just like with the digital filters.

Since TRP PLL is defined by three output variables TO_k, τ_k and T_k, the circuit is described by three transfer functions $H_{TO}(z)$, $H_\tau(z)$ and $H_T(z)$, given by eqs. (10), (11) and (12). Due to applicability of mat-lab tools for TRP PLL analyzes and due to three transfer functions, we are offered with very wide opportunities for the development of different filtering properties of TRP PLL. Apart from the selection of the parameters "a" and "m", we can choose one of three outputs which is the most suitable to meet the application requirements for TRP PLL, related to the frequency response of the circuit. Using mat-lab command "freqz", let us now present the magnitudes of the frequency responses of three transfer functions $H_{TO}(z)$, $H_\tau(z)$ and $H_T(z)$ in Fig. 6, for the same parameters a=0.6 and m=-2. We can notice in magnitudes of frequency responses that the filter corresponding to $H_T(z)$ is a low-pass filter, and that the filter which corresponds to $H_{TO}(z)$ is a band-pass filter. The filter of $H_\tau(z)$ behaves as a high-pass filter. In other word, TRP PLL possesses three outputs, whose filter characteristics are different. In the rough, one corresponds to a low-pass filter, another one functions as a band-pass filter and the third one represents a high-pass filter. This feature of TRP FLL offers the additional opportunities to meet the different filtering requirements.

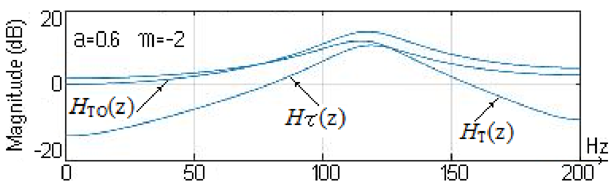

Fig.6. *Magnitudes of frequency responses of H_T (z), $H_{TO}(z)$ and $H_\tau(z)$ for a=0.6 and m=-2*

Let us now demonstrate the filtering abilities of TNP FLL for the noise suppression applications, which will be presented in the time domain. In dependence of the chosen values of the parameters "a" and "m", it can provide the powerful noise rejection. To demonstrate this, the simulation of its noise suppression ability is presented in Fig. 7. The input step, consisting of 10 time-units, which is strongly corrupted by uniform distributed noise, is denoted by "*TI*" in Fig. 7. The amplitude of noise is 10 t.u., peak to peak. It comes out from Fig. 7, that the noise suppression at the output periods will be better for the smaller values of the parameters "m" and "a", which must belong to the regain of the parameters for the stable PLL, shown in Fig. 2. Following this conclusion, three cases of the output periods TO_1, TO_2 and TO_3 are simulated for the different values of the parameter "m" and for a=0.01=const. For the output period TO_1, m=-0.7. For TO_2, m=-0.2 and for TO_3, m=-0.05. The noise suppression is apparent for TO_1. But it is considerable for TO_2 and extremely strong for TO_3. The smaller "m" provides better noise suppression. However if both parameters are very small, like in case for TO_3, the noise suppression is very powerful. However, it can also be seen from Fig. 7, that in case of better noise suppression, the transition time of TRP PLL is longer. In other word if the parameters are smaller, TNP PLL takes longer time to reach the stable state.

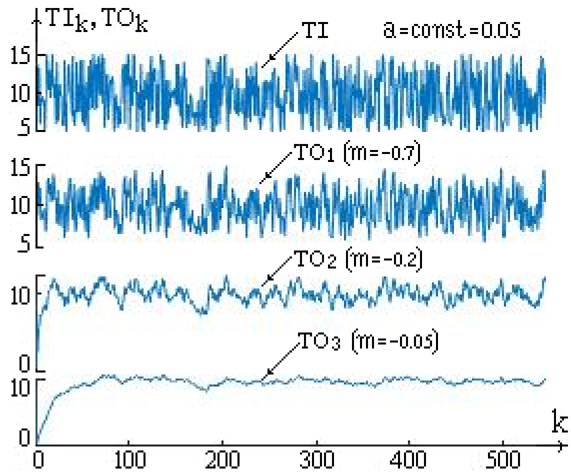

Fig.7. *Noise rejection ability of the circuit is presented in dependence on parameter "m" for a=0.05=constant.*

CONCLUSION

This TRP PLL represents an additional contribution to the recently described TRP PLL and TRP FLL, which are based on the processing of the input and output periods and the time differences between them. Due to these contributions, the field of Time Recursive Processing has got new weightiness and significance, offering good base for its further development.

It was also demonstrated that TRP PLL, choosing the corresponding system parameters, can provide very wide range of variety filter characteristics. Due to the more different outputs of the circuits and thanks to a wide choice of its functional and filter characteristics, TRP PLL offers good base for discovering of its new applications and for the development of new TRP PLL models, based on the same approach.

Although TRP PLL and the digital filter represent different types of systems, since the first one is based on the time processing and the other one is based on the processing of amplitudes, the article showed that mat-lab tools, devoted to the design of IIR digital filters, can be completely used for the development of TRP PLLs. Actually, TRP PLLs represent a special kind of digital filters. This contribution represents not only a great advance for theory of TRP PLLs. It is also an outstanding contribution to digital filter theory. Actually, this means that we could develop a new type of digital filter, designed specifically for impulse signals, that has many new features, compared to the existing approaches to the digital filters. The analysis of the features of the new approach to the digital filter, is very promising future work.

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