UNITECH 2018 GABROVO

INTERNATIONAL SCIENTIFIC CONFERENCE
16-17 November 2018, GABROVO

PID CONTROLLER WITH NOISE FILTER DESIGN BASED ON PSO OPTIMIZATION ALGORITHM–SISO SYSTEM WITH INTEGRATOR CASE

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Abstract
The PID controller with noise filter design using PSO optimization based on the frequency response of a process model Gm(jω), and maximization of the proportional gain, under constraints on the desired sensitivity to measurement noise, desired maximum sensitivity and desired maximum complementary sensitivity. The effectiveness of this method was illustrated by the simulation SISO process with integrator in the loop with the optimized PID controllers with the first-order and second-order noise filters. The set-point and load disturbance step responses are fast with small overshoot.

Keywords: PID control optimization, Noise filter, Measurement noise, Integrator, Robustness.

INTRODUCTION

For design PID controller with the different-order noise filter, based on optimization, a new effective optimization algorithm, named PSkO, was proposed under constraints on the sensitivity to measurement noise and robustness [1]. The PSkO algorithm is used here to analyze the impact of the noise filtering on the performance/robustness tradeoff. Algorithm and simulation results are presented.

EXPOSITION

The model of process $G_p(s)$, named $G_m(s)$, used in the present paper, is defined in [2] by the four parameters k_u, ω_u, φ, A,

$$G_m(s)=\frac{A\omega_u e^{-\tau s}/k_u}{s^2+\omega_u^2-A\omega_u e^{-\tau s}},$$
$$A=\frac{\omega_u k_u G_p(0)}{1+k_u G_p(0)},\ \tau=\frac{\varphi}{\omega_u} \qquad (1)$$

where φ is the angle of the tangent to the Nyquist curve $G_p(i\omega)$ at the ultimate frequency ω_u, and k_u is the ultimate gain of the process $G_p(s)$. The model $G_m(s)$ is an effective extension of the Ziegler-Nichols process dynamics characterization, guaranteeing an excellent approximation of the Nyquist curve $G_p(i\omega)$ in a large region around the ultimate frequency ω_u, presented in Fig. 1.

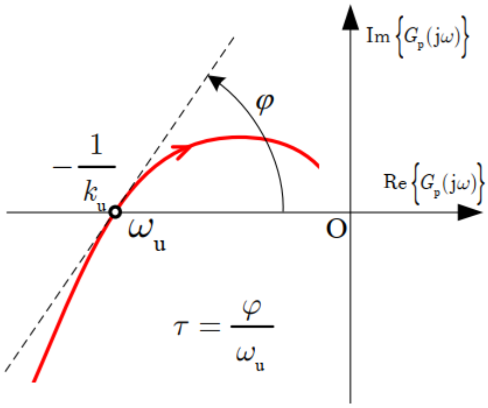

Fig. 1. *Illustration of tangent rule.*

The PID controller $C(s,q)$, are defined here by parameters $q=\{k,k_i,k_d,T_f\}$ and the following implementation

$$U(s)=k\big(bR(s)-Y_f(s)\big)+\frac{k_i}{s}\big(R(s)-Y_f(s)\big)-k_d sY_f(s)\ ,$$
$$Y_f(s)=F(s)Y(s) \qquad (2)$$

with b=0 if not stated otherwise. The LP filter $F(s)=F_n(s)$ is defined by the

$$F_n(s)=\frac{1}{(T_f s+1)^n},\ n=1,2 \qquad (3)$$

The optimal performance/robustness tradeoff is defined by the two performance indices, IAE and M_n, and by the two robustness indices, M_s and M_p. The load step disturbance and the IAE are defined by

$$Y_{\mathrm{d}}(s,q)=\frac{G_{\mathrm{x}}(s)}{1+C(s,q)G_{\mathrm{x}}(s)}\frac{1}{s}$$

$$IAE(q)=\int_0^{\infty}\left|y_{\mathrm{d}}(t,q)\right|dt \qquad (4)$$

The effect of the load step disturbance is defined by the index [3]

$$J_{\mathrm{d}}(q)=\max_{\omega}\left|Y_{\mathrm{d}}(\mathrm{i}\omega,q)\right| \qquad (5)$$

For the loop-transfer function defined by $C_{\mathrm{L}}(s)$,parameters M_{s} and M_{p}, and if the first-order noise filter is applied in the PID controller, parameter $M_{\mathrm{nz}}=M_{\mathrm{n}\infty}$, are given by:

$$C_{\mathrm{L}}(i\omega,q)=C(i\omega,q)G_m(i\omega)$$

$$S(i\omega,q)=\frac{1}{1+C_{\mathrm{L}}(i\omega,q)} \qquad (6)$$

$$M_S=\max_{\omega}\left|S(i\omega,q)\right|,\ M_p=\max_{\omega}\left|1-S(i\omega,q)\right| \qquad (7)$$

$$M_{\mathrm{n}}=\max_{\omega}\left|C(\mathrm{i}\omega,q)S(\mathrm{i}\omega,q)\right| \qquad (8)$$

For the PID controller $C(i\omega,q)$ with first-order noise filter $F(s)$, M_{n} can be defined as the sensitivity to the high frequency measurement noise

$$M_{\mathrm{n}\infty}=lim_{s\to\infty}C(s,q) \qquad (9)$$

But, $M_{\mathrm{n}\infty}=0$ when the second-order or higher order noise filter $F(s)$ is applied in the PID controller (2). In this case, the sensitivity to measurement noise $M_{\mathrm{n}}=M_{\mathrm{n}2}$ is defined by the integral

$$M_{n2}=\sqrt{\frac{1}{\omega_C}\int_{\varepsilon}^{\omega_c}C_{\mathrm{u}}(\mathrm{i}\omega,q)C_{\mathrm{u}}(-\mathrm{i}\omega,q)d\omega}\ ,$$

$$C_u(i\omega,q)=C(i\omega,q)S(i\omega,q) \qquad (10)$$

Measurement noise can be modelled as band-limited white noise $n_{\mathrm{w}}(t)$ with the cutoff frequency ω_{c}. The noise $n_{\mathrm{w}}(t)$ is obtained from the Band-Limited white noise generator defined by the "noise power" b_{w}=PSD and sample time T_{s}, discussed in detail in Appendix. The cutoff frequency ω_{c} of this band-limited white noise is defined by

$$\omega_C=\frac{\pi}{T_s},\ T_s\approx\frac{T_{\mathrm{f0}}}{N_{\mathrm{C}}},\ N_{\mathrm{C}}=2 \qquad (11)$$

where T_{f0} is the value of the noise filter time constant in the PID controller. Sensitivity to measurement noise $M_{\mathrm{n}2}$ can be interpreted as

$$M_{\mathbf{n2}}=\sqrt{\sigma_{\mathbf{u}}^2/\sigma_{\mathbf{n}}^2} \qquad (12)$$

where $\sigma_{\mathbf{u}}^2$ i $\sigma_{\mathbf{n}}^2$ are variances of controler output signal and measurement white noise on band-limited frequency range[1, Appendix A].

Sensitivity $M_{\mathrm{n}2}$ in (9) is calculating using numerical integration with ε=0.001.

PID controller design based on PSkO optimization algorithm is defined by [1]

$$q=\arg\left\{\min_q\left(\frac{1}{|k|}+\lambda_0\sum_1^4\psi_i(\chi_i-\chi_{id})\right)\right\} \qquad (13)$$

$$\psi_{\mathrm{i}}(\chi_{\mathrm{i}}-\chi_{\mathrm{id}})=\begin{cases}0 & for\ \chi_{\mathrm{i}}\le\chi_{\mathrm{id}} & i=1,2,3\\ 0 & for\ 1<\chi_{\mathrm{i}}<1.01 & i=4\\ \chi_{\mathrm{i}}-\chi_{\mathrm{id}} & for\ \chi_{\mathrm{id}}<\chi_{\mathrm{i}} & i=1,2,3\\ 1 & \chi_{\mathrm{i}}\le 1\ or\ 1.01\le\chi_{\mathrm{i}} & i=4\end{cases} \qquad (14)$$

where $G_{\mathrm{p}}(i\omega)$ is the frequency response of the process, $\lambda_0=10^{10}$, $\chi_1=M_{\mathrm{s}}$, $\chi_{1\mathrm{d}}=M_{\mathrm{sd}}$, $\chi_2=M_{\mathrm{p}}$, $\chi_{2\mathrm{d}}=M_{\mathrm{pd}}$, $\chi_3=M_{\mathrm{n}2}$, $\chi_{3\mathrm{d}}=M_{\mathrm{n2d}}$, and $|Y_{\mathrm{d}}(\mathrm{i}\omega,q)|=|G_{\mathrm{p}}(\mathrm{i}\omega)S(i\omega,q)/\mathrm{i}\omega|$. Where with subscript d is denoted the desired values of M_{n2d}, M_{sd} and M_{pd}. Parameters defined in [1, Appendix B] are applied as initial values $q_0=\{k_0, k_{\mathrm{i}0}, k_{\mathrm{d}0}, T_{\mathrm{f}0}\}$ for optimization (12)-(13), and define $\mathrm{PID}_{\mathrm{tun}}$ controller with first-order noise filter. From $\mathrm{PID}_{\mathrm{tun}}$ controller increasing filter order and increasing filter time constant T_{f} in (3) we reduce the control signal (controller output signal) activity to the desired level.

An good performance measure about a reduction of the control signal activity is obtained by the visual inspection of the control signal derivatives, defined by $\Delta\mathrm{u}(k)=\{u(kT_{\mathrm{s}})-u((k-1)T_{\mathrm{s}})/T_{\mathrm{s}}$, for the sample time T_{s} and $k=1,2,\ldots,K$, $K=T_{\mathrm{sim}}/T_{\mathrm{s}}$, where T_{sim} is the time of simulation. Another measure of control signal activity is the variance of the difference of control signal $\Delta u(k)=\mathrm{u}(kT_{\mathrm{s}})-u((k-1)T_{\mathrm{s}})$ is defined by $\sigma_{\Delta u}^2=\Sigma_{k=1}^K(\Delta u(k))^2/K$ for $K=T_{\mathrm{sim}}/T_{\mathrm{s}}$. When the desired reduction of the control signal activity is obtained, the desired M_{n2d} is calculated and defined together with the desired values M_{sd} and M_{pd}.

Next, we applied PSkO constrained optimization procedure (13)-(14) to obtain final $\mathrm{optPID}_{\mathrm{f}}$ controller.

The proposed procedure is analyzed here by simulation of the integrating $G_{\mathrm{p}2}(s)$ processes $G_{\mathrm{p}}(s)$. The satisfactory reduction of the control signal activity is obtained for the $\mathrm{PID}_{\mathrm{f}}$ controller, with $n=2$, defined by gains k_0, $k_{\mathrm{i}0}$, $k_{\mathrm{d}0}$ and $T_{\mathrm{f}}=gT_{\mathrm{f}0}$, $g=1$. Further improvement a performance of control system was obtained applying PSkO constrained optimization procedure. Additionally reduction of control signal activity is possible with increasing parameter g, i.e. filter time constant, but this leads to deterioration other performance measures of control system, like speed of response to reference step and elimination of load step disturbance. This is clearly presented in Table 1 on a next page.

It is important to highlight here a fact that constrains in PSkO optimization procedure have clear physical interpretation. This is also important form engineering point of view. If designer want fast rejection of the load disturbance signal and fast tracking of reference signal, then designer set higher values of M_{sd} and M_{pd}. If greater variation of the dead-time or the process gain is expected, the designer will specify the desired performance with lower values of M_{sd} and M_{pd}, to obtain a robust tuning. If designer set higher values of $M_{n2d}(M_{n\infty d})$, then result is faster rejection of the load disturbance and higher control signal activity, and vice versa. If the actuator requires the small control signal variation, the PID controller with n=2 must be applied. PID with small filter time constant and low filter order can tolerate significant control signal activity.

SIMULATION ANALYSIS

In this section the set-point and load disturbance step responses of integrating processes $G_p(s)$ and the corresponding models model $G_m(s)$, in the loop with different type of controllers, are compared. It is demonstrated that almost the same performance/robustness tradeoff is obtained if the model $G_m(s)$ is used instead of the process $G_p(s)$. The integrative process is analyzed

$$G_p(s) = \frac{e^{-0.5s}}{s(1.2s+1)^3} \tag{15}$$

Parameters defining the model (15) are given by: k_u=0.5640, ω_u=0.4080, τ=1.7647, A=0.4080. PID$_{tun}$ controller parameters was obtained applying tuning formulae in [1]. PID$_f$ controller parameters are same as PID$_{tun}$ except filter order is increased to n=2, in order to reduce controller output signal variance. Finally, PSkO algorithm was applied, with initial values of PID controller parameters in previous case, and optPID$_f$ controller was obtained. Performance/robustness indices and parameters of PID controllers are presented in Table 1.

CONCLUSION

In this paper we analyzed proposed PID controller with filter noise design. In order to reduce control signal activity, with small degradation performance/robustness trade-off, design goal was to find a parameters of regulator and filter noise order and time constant. Design is based on Particle Swarm Optimization. Reducing a control signal activity will reduce actuator wear a prolong service intervals for actuator change. In order to further reducing a control signal activity. Research efforts in future investigation will be implementation a more complex noise filters, with small degradation of control system performance/robustness. Fig.2 represents response and control signal derivatives of three proposed PID controllers.

ACKNOWLEDGEMENTS

A.D. Micić gratefully acknowledge the financial support from Serbian Ministry of Science and Technology (Project III 47016).

Table 1. Parameters and performance/robustness tradeoff obtained by the PID controllers used to obtain closed-loop responses in Fig. 2 , PSD=3e-4, and b=0 in (2).

$G_p(s)$/PID	n	T_s	$M_{n\infty}$	M_{n2}	M_s	M_p	IAE	k	k_i	k_d	T_f
G_p/PID$_{tun}$	1	0.1	3.36	-	2.23	1.62	32.2	0.3712	0.0313	0.8240	0.2451
G_p/PID$_f$	2	0.1	0	1.08	2.55	1.91	32.1	0.3712	0.0313	0.8240	0.2451
G_p/optPID$_f$	2	0.1	0	1.28	1.99	1.56	39.7	0.3008	0.0305	0.7878	0.2062

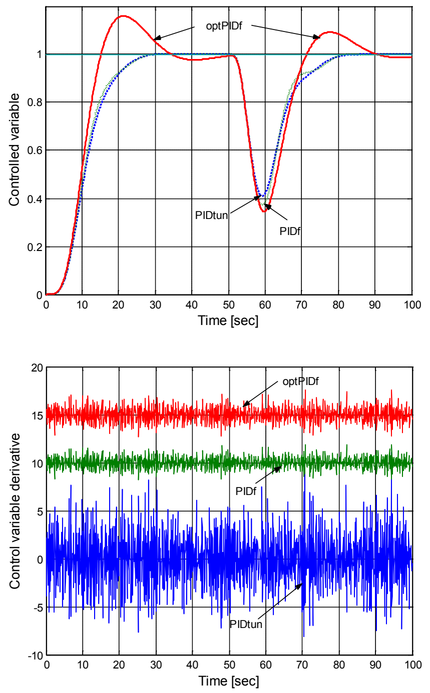

Fig. 2. Responses of the integrating process $G_p(s)$ in the loop with PID_{tun}, PID_f, $optPID_f$ in Table 1: a) Set-point and load step disturbances, the load step D(s)= -0.2exp(-50s)/s; b) control signal derivative (biases are inserted for PID_f, $ptPID_f$), PSD=0.0003 and T_s=0.1.

REFERENCE

[1] A. D. Micić, M.R. Mataušek, “Optimization of PID controller with higher-order noise filter”, *Journal of Process Control*, Vol.24, Issue 5, May 2014, pp 694-700.

[2]T.B. Šekara and M.R. Mataušek, “Revisiting the Ziegler-Nichols process dynamics characterization”, J. Process Control Vol. 20, Issue 3, pp. 360-363, 2010.

[3] Kristiansson, B. Lennartson, Evaluation and simple tuning of PID controllers with high-frequency robustness, J. Process Control 2006; 16:91-102.