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THE TOPSIS METHOD FOR EXPERIMENTAL COMPARISON OF MULTIPLE METAHEURISTIC ALGORITHMS OVER A SET OF PROBLEMS: A CASE STUDY OF THE MULTI-CASE CEED PROBLEM

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Abstract

In recent years, there has been a growing interest in developing new metaheuristic algorithms for optimizing multiple-problems. Due to the stochastic nature of algorithms, their solutions and behaviors are different for different single-problems contained in one multiple-problem. Therefore, in many cases, it is difficult to choose the best algorithm that will solve a specific multiple-problem. This paper uses the multi-criteria decision-making (MCDM) method to rank the proposed algorithms for solving concrete multiple-problem. We take the multi-case Combined economic emission (CEED) problem as a case study. Specifically, we rank five proposed algorithms according to different performance measures using the TOPSIS method to solve four CEED problem cases. Additionally, the obtained results were validated using the EDAS method, confirming the final rank of the analyzed algorithms.

Keywords: metaheuristics, CEED, MCDM methods, TOPSIS method, EDAS method.

INTRODUCTION

In recent years, a large number of metaheuristic algorithms have been proposed to solve various optimization problems. These problems are often multiple, i.e. they contain several sub-problems represented by different objective functions, so the proposed algorithm can be good for some sub-problems but slightly weaker for others. This is consistent with the "No free lunch" theorem [1], which shows that no algorithm is better than others for any problem. Due to the stochastic nature of metaheuristic algorithms, performance measures such as best and mean value, standard deviation, error rate, computation time, convergence, etc., are used to evaluate their efficiency and effectiveness. These performance measures differ for different algorithms and problems, so it is often difficult to evaluate which algorithm from the many proposed in the literature is the most acceptable for solving a certain multiple-problem. In this paper, we propose multi-criteria decision-making (MCDM) methods for selecting the most acceptable algorithm from the set of proposed ones for solving a multiple-problem. As a case study, we took the multiple-problem Combined economic and emission dispatch (CEED) problem, for the solution of which a large number of algorithms have been proposed in the literature. The MCDM methods we apply are the TOPSIS and EDAS methods. The algorithms we evaluate are AWDO [2], FA [3], MSA [4], PSOGSA [5], and PSO [6].

TESTING THE ALGORITHMS

We test algorithms on a standard IEEE 30-bus 6-generator system with a total load demand of 283.4 MW. CEED is the adjustment of the output power of several generators in a thermal power plant to

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minimize fuel cost and/or emission of toxic gases by satisfying the constraints in the system. The most common objective functions (*f*) in this optimization are the following:

$$f_1 = \sum_{g\in G} F(P_g) + \gamma \sum_{g\in G} E(P_g),\ g = 1,\ 2,\ldots,G$$
$$f_2 = \sum_{g\in G} F(P_g) + \sum_{g\in G} \left| d_g \sin\left(e_g \left(P_g^{\min} - P_g\right)\right)\right|,\ g = 1,\ 2,\ldots,G \quad (1)$$
$$f_3 = \sum_{g\in G} E(P_g),\ g = 1,\ 2,\ldots,G$$
$$f_4 = \sum_{g\in G} F(P_g) + \gamma \sum_{g\in G} E(P_g) + \sum_{g\in G} \left| d_g \sin\left(e_g \left(P_g^{\min} - P_g\right)\right)\right|,\ g = 1,\ 2,\ldots,G$$

where f_1 is an objective function that minimizes the sum of fuel costs and emissions in the power plant, simultaneously; f_2 is a function that minimizes the sum of fuel costs taking into account the valve point effect in the thermal power plant; f_3 is a function that minimizes the total emission in the power plant; *G* is the total number of generators under consideration; f_4 is an objective function that minimizes the sum of fuel costs and emissions in the power plant, simultaneously taking into account the valve point effect; P_g (MW) is the output power of the generator *g*; $P_g^{\min}$ (MW) is the minimum power of the generator *g*; d_g and e_g are the coefficients of valve point effect in the thermal station; γ is the scaling factor; $F(P_g)$ and $E(P_g)$ are functions of fuel cost ($/h) and emission (t/h) respectively, dependent on the output power of the generator *g*. The forms of these functions are as follows:

$$F_g\left(P_g\right) = a_g + b_g P_g + c_g P_g^2 \quad (2)$$

$$E_g\left(P_g\right) = \alpha_g + \beta_g P_g + \eta_g P_g^2 + \xi_g \exp\left(\lambda_g P_g\right) \quad (3)$$

where a_g , b_g and c_g are the fuel cost coefficients of the generator *g*; α_g, β_g, η_g, ξ_g and λ_g are the emission coefficients of the generator *g*. The constraints we applied in solving this problem are:

- the constraint of the generator's power,

$$P_g^{\min} \le P_g \le P_g^{\max} \quad (4)$$

- the power balance in the system,

$$\sum_{g\in G} P_g - P_D - P_{loss} = 0, \quad (5)$$

where $P_g^{\min}$ and $P_g^{\max}$ are the minimum and maximum power of generator *g*, respectively; P_D is the total power of the consumer, P_{loss} is the power loss in the transmission system. In order to maintain the balance condition during the calculation, the power of one of the generators (slack generator) is calculated from (5) at each iteration. Power loss in the transmission system, P_{loss}, is expressed from the output powers of generators according to Kron's formula, as follows:

$$P_{loss} = \sum_{g\in G} \sum_{j\in G} P_g B_{gj} P_j + \sum_{g\in G} B_{0g} P_g + B_{00} \quad (6)$$

where B_{gj} and B_{0g} are the coefficients of the ***B**-loss* matrix, and B_{00} is a constant. Coefficients of fuel cost, emission and *B-loss* matrices are taken in this paper from [7]. The algorithms are implemented in MATLAB R2017a computational environment and run on 1.3 GHz, with 8.0 GB RAM. The best results of the simulations are obtained after 30 runs. The general structure of algorithms for solving the CEED problem consists of the following steps:

Step 0: Preparation	Choose the number of search agents (solution candidate) $N \geq 2$ in the search space, the coefficients of the algorithm, the maximum number of iterations t_{max} and the fitness function $f(x)$ from (1). Set the iteration counter to $t = 0$.
Step 1: Initialization	Generate a random population of N search agents. The initial positions of each agent $x_i^{(0)}$ are randomly selected between minimum and maximum values of the control variables (i.e., power outputs of generators). Calculate the power of the slack generator for each agent. Calculate: $f^* = \min\left\{ f\left(x_i^{(0)}\right),\ i = 1,\ \ldots,\ N \right\}, \quad x^* = \arg f^*.$
Step 2: Update $x_i^{(t)}$	Generate a new population of agents $x_i^{(t+1)}$, by applying an algorithmic operator to each agent from the current population. Calculate the power of the slack generator. Calculate: $f_{min} = min\left\{ f\left(x_i^{(t+1)}\right),\ i = 1,\ \ldots,\ N \right\}.$ If $f_{\min} < f^*$, put $f^* = f_{\min}$ and $x^* = \arg f_{\min}$.
Step 3: End	If $t = t_{\max}$, stop the algorithm. Otherwise, set $t = t + 1$ and return to step 2. Adopt the solution x^* as the final approximate solution to the problem.

The coefficients of the algorithms are shown in Table 1.

Table 1. Coefficients of the algorithms applied to the test system.

AWDO			FA					MSA			PSOGSA						PSO			
N	t_{max}	α ,g, RT, c	N	t_{max}	α	$\beta_{\min}$	γ	N	t_{max}	N_c	N	t_{max}	G_0	α	C_1	C_2	N	t_{max}	C_1	C_2
50	200	optimized	50	200	0.25	0.20	1	50	200	6	50	200	1	20	0.5	1.5	50	200	0.5	1.5

Table 2 shows the minimum, mean, standard deviation, error rate, convergence, and computation time values for cases of appliing AWDO, FA, MSA, PSOGSA, and PSO to the test system are shown in Table 2. From the results, it is evident that the minimum values of the fuel cost and emission, and fuel cost and emission simultaneously, are the same or close to each other for all four algorithms. Other compared values are close to each other or significantly different. In the next chapter, algorithms are ranked based on their performance measures across different functions.

RANKING ALGORITHMS USING THE TOPSIS METHOD

Based on the findings from the previous phase of the research, it can be concluded that there was no significant difference in the performance of the five tested algorithms (AWDO, FA, MSA, PSOGSA, PSO) when addressing CEED problems across all functions (variants) simultaneously. Consequently, alongside evaluating the metaheuristic algorithms, a multi-criteria decision-making method was employed to rank the algorithms based on their performance across different functions. The top-ranked algorithm for solving the CEED problem was identified by considering various factors, including the best results, standard deviation (SD), mean value, error rates, computation time, and convergence across the individual problem variants. The TOPSIS method was utilized to find the best solution.

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) emerged in the 1980s as a method for multi-criteria decision-making. It identifies the best alternative by measuring the shortest Euclidean distance to the ideal solution while maximizing the distance from the negative ideal solution [8], [9], [10], [11]. Alternatives are ranked based on an overall index calculated from the distances to the perfect solutions. This MCDM method is widely employed to address various decision problems [12], [13], [14]. The initial data for a multicriteria problem can be found in Table 2.

Table 2. The initial data for ranking tested algorithms

Criteria →	f_1	f_2	f_3	f_4	f_1	f_2	f_3	f_4
Min/Max	min	min	min	min	min	min	min	min
Weight	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
↓Alternatives / Scenario→	**Best**				**SD**			
AWDO	815.82291063	635.92045215	0.194178511	861.70746581	0.039554634	6.861898931	2.01239E-14	0.522555834
FA	815.82291064	635.85555344	0.194178511	861.70728467	1.69099E-07	4.142287984	1.27759E-09	0.788621003
MSA	815.82291133	635.88017914	0.194178512	861.70997504	4.20478E-06	10.48924953	2.11788E-09	0.684718756
PSOGSA	815.82291063	635.82011047	0.194178511	861.70505749	15.79001237	15.57652466	0.007090474	24.02610905
PSO	815.82291063	635.90446527	0.194178511	861.70280274	6.658129574	8.401397207	0.002638085	7.55054294
↓Alternatives / Scenario→	**Mean value**				**Error rate**			
AWDO	815.83013229	640.44711163	0.194178511	862.03819713	0.000885199	0.727721738	3.7164E-12	0.038922281
FA	815.82291080	637.95788593	0.194178511	862.26358454	2.074E-08	0.336223316	1.71104E-07	0.06507833
MSA	815.82291765	645.42733709	0.194178514	862.56767428	8.60208E-07	1.510997601	1.42011E-06	0.100367729
PSOGSA	832.13481841	661.44054499	0.204305302	891.78349992	1.999442228	4.029509934	5.215196531	3.490843604
PSO	817.99498404	648.73761870	0.194660157	866.21645830	0.266243248	2.031629391	0.248043051	0.52380653
↓Alternatives / Scenario→	**Computation time**				**Convergence**			
AWDO	3.0762	4.0517	3.5368	3.5821	68	171	37	39
FA	4.22280	2.97690	2.49058	2.53068	61	153	37	139
MSA	3.98072	3.19908	4.74918	2.86923	127	178	77	199
PSOGSA	1.6691	1.5170	1.6088	1.6975	28	52	29	46
PSO	3.49220	3.05458	4.24198	3.28438	43	156	51	60

The best-ranked algorithm applied in four different functions according to the best results, computation time and convergence is PSOGSA, followed by the FA algorithm in the case of SD and mean value. The obtained results are shown in the Table 3.

Table 3. TOPSIS results of the complete ranking of the analyzed algorithms

	Best		**SD**		**Mean value**	
	C_i	*Rank*	C_i	*Rank*	C_i	*Rank*
AWDO	0,44790	4	0,86988	2	0,94070	2
FA	0,62500	2	0,99005	1	0,99638	1
MSA	0,21743	5	0,72108	3	0,83152	3
PSOGSA	0,83179	1	0,00000	5	0,00000	5
PSO	0,61169	3	0,63144	4	0,73194	4
	Error rate		**Time**		**Convergence**	
	C_i	*Rank*	C_i	*Rank*	C_i	*Rank*
AWDO	0,96196	1	0,26780	3	0,47776	3
FA	0,96009	2	0,42400	2	0,44803	4
MSA	0,82831	3	0,24167	5	0,0000	5
PSOGSA	0,00000	5	1,00000	1	0,98200	1
PSO	0,74008	4	0,26592	4	0,49360	2

Additionally, the validation of obtained results was achieved using the EDAS method. This multi-criteria decision-making method is used very often in comparative analysis with the TOPSIS method, as evidenced by numerous studies [15], [16]. The final rank of the analyzed algorithms using the EDAS method is shown in Table 4.

Table 4. EDAS results of the complete ranking of the analyzed algorithms

	Best		**SD**		**Mean value**	
	S_i	*Rank*	S_i	*Rank*	S_i	*Rank*
AWDO	0,00001	4	0,470102	4	0,451004	3
FA	0,18032	2	0,598036	2	0,662027	1
MSA	0,00000	5	0,890626	1	0,655629	2
PSOGSA	0,97600	1	0,488662	3	0,317656	4
PSO	0,02400	3	0,011338	5	0,182344	5
	Error rate		**Time**		**Convergence**	
	S_i	*Rank*	S_i	*Rank*	S_i	*Rank*
AWDO	0,475452	3	0,017072	3	0,189223	2
FA	0,636795	2	0,091329	2	0,063378	4
MSA	0,648713	1	0	4	0	5
PSOGSA	0,285508	4	1	1	0,828385	1
PSO	0,214492	5	0	4	0,171615	3

The results obtained by the ranked algorithms using the EDAS method showed consistency in the obtained ranks of different performance measures compared to the TOPSIS method results. Namely, the application of the EDAS method indicates that the best-ranked algorithm is PSOGSA (according to the best results, computation time, and convergence), which points out that the validity of the obtained results was achieved.

CONCLUSION

A large number of metaheuristic algorithms for solving certain optimization problems have been proposed in the published literature. Problems that are solved using these algorithms can be multiple-problems, i.e., they can have more variants and contain more sub-problems. Therefore, according to the "No free lunch" theorem, one algorithm may not be the best for solving all variants of the problem for which it was proposed. Some of the proposed algorithms are better than others when solving one number of variants, and some when solving other variants. In this paper, the MCDM method TOPSIS is proposed, by means of which among the proposed algorithms for solving a multiple-problem, one can be selected as the most acceptable. Also, using this method, algorithms can be ranked according to the performance measure that is chosen as the ranking criterion. The proposed procedure can help decision-makers to choose the most acceptable algorithm for solving their problem or a part of that problem from the multitude of proposed algorithms.

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