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USAGE OF FREQUENCY LOCKED LOOP FOR TIME-DIGITAL FILTERING AND OTHER APPLICATIONS

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Abstract

This article describes development, analysis, application, and simulation of a recursive Time Frequency Locked Loop (TFLL) based on the measurement and processing of the periods of the input and output signals. TFLL is a linear discrete system of the second order, which regulates its output once per the input period. The parameters of TFLL are determined by the ratios of clock frequencies which have to be in the predefined relationships for the stable functioning of TFLL. Mathematical description, analysis of stability conditions and properties of TFLL are performed using Z transform. The relations of the parameters which correspond to the specific applications are analyzed. Using mathematical analyses and simulations, it is shown that TFLL is suitable for powerful noise rejection, for the different predicting and tracking applications, for the measurements of the frequency, for the filtering of impulse signal periods as well as for the other usual applications of FLL. Special emphasis is given to the development of a Time digital filter based on TFLL, using the theory of digital filters and the Mat-lab tools intended for digital filters.

Keywords: Digital circuits, Frequency locked loops, Phase locked loops, Digital filters, Linear discrete system.

INTRODUCTION

Time Frequency Locked Loop (TFLL), Time Phase Locked Loop (TPLL) and Time digital filter are based on the processing of the periods of the input and output impulse signals and time differences between them. They are recently described in refs. [1 to 12]. They represent one fundamentally new approach in the scientific literature from the point of view of constructions, descriptions, way of signal processing, way of analyzes and applications. The applications of these systems are numerous. In addition to digital filtering of the pulse signal period, they are applied in the field of tracking and prediction, phase and time shifting, frequency multipliers, frequency synthesizers, noise rejections, averaging of the periods, frequency measurement and the others.

In this article, we will present the full recursive second-order TFLL model, perform various analyses in the time and frequency domain, make simulations and describe the development of Time digital filter, based on TFLL. In addition, we will demonstrate some of its applications.

The articles and books [13-24] are used as the theoretical and mathematical base.

DESCRIPTION AND ANALYSIS OF FLL

The general case of the input and output signals Sin and Sop for TFLL of M-th order, is shown in Fig. 1. Periods TI_k and TO_k, as well as the time differences τ_k, occur at discrete times t_k, t_{k+1}, t_{k+2},…t_{k+M-1}, t_{k+M}, which are defined by the falling edges of the pulses of Sop. The difference equations of full second-order $TFLL_2$ are presented in eq. (1), where

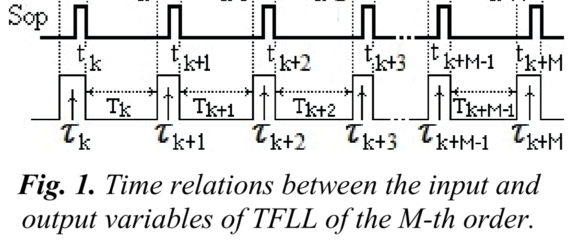

Fig. 1. *Time relations between the input and output variables of TFLL of the M-th order.*

b_1, b_2, a_1 and a_2 are the system parameters of FLL_2. One additional natural relation between the time variables, which yields from Fig. 1, is shown in eq. (2). In order to found the transfer functions of $TFLL_2$ let us find the Z transforms of eqs. (1) and (2). They are presented in eqs. (3) and (4). If we calculate TO(z) from eq. (3), we can after that substitute $TO_1=b_1TI_0+a_1TO_0$ into TO(z) and get the final expression for TO(z), given by eq. (5). Note that the previous expression for TO_1 comes out from eq. (1), for k = -1. If we substitute now TO(z) from eq. (5) into eq. (4), we can found out the expression for τ(z), shown in eq. (6). Two transfer functions describing $TFLL_2$, which are given by (7) and (8), can be defined from (5) and (6). Note that TO_0, TI_0 and τ_0 in eqs. (3) and (4) are the initial conditions of TO_k, TI_k and τ_k.

$$TO_{k+2} = b_1 \cdot TI_{k+1} + b_2 \cdot TI_k + a_1 \cdot TO_{k+1} + a_2 \cdot TO_k \quad (1)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \quad (2)$$

$$\begin{aligned} z^2TO(z) - zTO_1 - z^2TO_0 &= z \cdot b_1 \cdot TI(z) - z \cdot b_1 \cdot TI_0 + \\ b_2 \cdot TI(z) + z \cdot a_1 \cdot TO(z) &- z \cdot a_1 \cdot TO_0 + a_2 \cdot TO(z) \end{aligned} \quad (3)$$

$$z \cdot \tau(z) - z \cdot \tau_0 = \tau(z) + TO(z) - TI(z) \quad (4)$$

$$TO(z) = TI(z)\frac{z \cdot b1 + b2}{z^2 - z \cdot a1 - a2} + z\frac{z \cdot TO_0}{z^2 - z \cdot a1 - a2} \quad (5)$$

$$\begin{aligned} \tau(z) &= TI(z)\frac{-z-(b2+a2)}{z^2 - z \cdot a1 - a2} + \\ &\frac{z}{z-1}\frac{z \cdot TO_0}{z^2 - z \cdot a1 - a2} + \frac{z \cdot \tau_0}{z-1} \end{aligned} \quad (6)$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{z \cdot b1 + b2}{z^2 - z \cdot a1 - a2} \quad (7)$$

$$H_{\tau}(z) = \frac{\tau(z)}{TI(z)} = \frac{-z-(b2+a2)}{z^2 - z \cdot a1 - a2} \quad (8)$$

In order to analyze $TFLL_2$, let us suppose that the step function TI(k)=TI=constant is applied to the input. Z transform of TI(k) is TI(z)=TI·z/(z-1). If we enter TI(z) into eq. (5), using the final value theorem, it is possible to find TO_∞=limTO(k) if $k \to \infty$, using TO(z). This is shown in eq. (9). It comes out from eq. (9) that $TFLL_2$ will be functional if eq. (10) is satisfied. Only in this case TO_∞=TI.

$$TO_\infty = \lim[(z-1) \cdot TO(z)]_{z\to 1} = TI \cdot \frac{b1 + b2}{1 - a1 - a2} \quad (9)$$

$$a1 + a2 + b1 + b2 = 1 \quad (10)$$

In the same way, the final value of *τ(k)* if $k \to \infty$ can be determined. Providing that the step function TI(k)=TI is applied to the input, Tl(z) in eq. (6) should be substituted by TI·z/(z-1). We can find out the final value $\tau_\infty = \lim\,[\tau(k)]_{k\to\infty}$, using the final value theorem in Z transform notation $\tau_\infty = \lim[(z-1)\cdot\tau(z)]_{z\to 1}$. Applying this expression, we can get τ_∞, shown in eq. (11). As we can see from eq. (11), time difference τ_∞ in the stable state of $TFLL_2$ depends on the initial conditions τ_0 and TO_0, as well as on the system parameters and constant input period TI. That means $TFLL_2$ does not possess the properties of a PLL.

$$\tau_\infty = -TI\frac{b2 + a2 + 1}{1 - a1 - a2} + \frac{TO_0}{1 - a1 - a2} + \tau_0 \quad (11)$$

All the previous conclusions, including the results given by eqs. (9), (10) and (11), are valid only if the system is stable. $TFLL_2$ is the stable system if the poles $|z_1| < 1$ and $|z_2| < 1$, where z_1 and z_2 are the zeros of the polynomial $z^2 - z \cdot a_1 - a_2$ in eq. (7) or in eq. (8). The zeros z_1 and z_2 are shown in eq. (12).

$$z_{1/2} = \frac{a_1}{2} \pm \sqrt{\left(\frac{a_1}{2}\right)^2 + a_2} \quad (12)$$

The conditions $|z_1| < 1$ and $|z_2| < 1$ define the region in the plane of parameters b_1 and b_2, where TFLL is the stable system. This region, shown in Fig. 2, is located between three mathematical straight lines defined by $a_2=-1$, $a_2=a_1+1$ and $a_2=-a_1+1$.

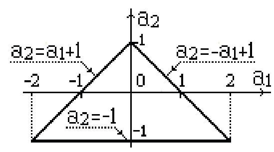

Fig. 2. *Figure shows the region of a_1 and a_2 for the stable $TFLL_2$.*

In order to investigate the tracking performances of $TFLL_2$ we will analyze the behavior of TO_k and τ_k for the ramp input. All time variable will be expressed in time units (t.u.). Note that t.u. can be µs, ms, or any other time unit, assuming the same time unit for all time variables. Because of simplicity, "t.u." units are omitted from the diagrams.

Let us denote the tracking error of TO_k by K_{TOV} and the final value of time difference τ_k by $\tau_{V\infty}$, where "V" denotes that the input period is a velocity function $TI_V(k)=c\cdot k$ and "c" is the constant. Using the final value theorem, K_{TOV} and $\tau_{V\infty}$ can be expressed by Z transform notation, as in eqs. (13) and (14). If we enter $Z[TI_V(k)]=TI_V(z)=z\cdot c/(z-1)^2$ into eq. (13), we will get K_{VTO}, given by eq. (15). It is obvious that K_{VTO} can be equal to zero only if eq. (16) is satisfied. In order to calculate $\tau_{V\infty}$ we will first substitute $b_2+a_2=-1$ into (6), that is, $\tau_V(z)$ will get the simplified form. If we after that enter $\tau_V(z)$ in (14), we can calculate $\tau_{V\infty}$, shown in eq. (17).

$$K_{TOV}=\lim\{(z-1)\cdot[TO(z)-TI_V(z)]\}_{z\to 1} \quad (13)$$

$$\tau_{V\infty}=\lim[(z-1)\cdot\tau_V(z)]_{z\to 1} \quad (14)$$

$$K_{VTO}=c\cdot\frac{b_2+a_2+1}{a_1+a_2-1} \quad (15)$$

$$b_2+a_2=-1 \quad (16)$$

$$\tau_{V\infty}=\frac{-c}{1-a1-a2}+\frac{TO_0}{1-a1-a2}+\tau_0 \quad (17)$$

Let us now analyze the abilities of $TFLL_2$ for the tracking of the velocity input (ramp) using the simulations. The simulations of TO_k, K_{VTOk}, and τ_{Vk}, for $TI_k=20+5\cdot k$ [t.u.] is shown in Fig. 3. The simulation is made for three combinations of the system parameters and the initial conditions, which are presented in Fig. 3. Note that the first and the second combinations of parameters b_1, b_2, a_1, and a_2, signed by "1" and "2" satisfy both, eq. (16) and eq. (10). Due to this fact, the output periods of TO_{1k} and TO_{2k} track TI_{Vk} without error. The corresponding errors K_{VTO1k} and K_{VTO2k} tend to zero. Since $K_{VTO1}=K_{VTO2}=0$, the simulation results agree with eq. (15). Note also that, for the first two combinations of parameters, the corresponding τ_{1k} and τ_{2k} tend to respectively $\tau_{V1\infty}$ and $\tau_{V2\infty}$. According to eq. (17), $\tau_{V1\infty}$=-10.43 and $\tau_{V2\infty}$=38.50. After only eight steps τ_{1k}=-10.51 and τ_{2k}=38.48, proving so the correctness of eq. (17). The third combination of parameters, signed by "3" in Fig. 3, satisfies eq. (10), but it does not satisfy eq. (16). That means TFLL is the stable system but it is not adapted to track the ramp without error. We can see in Fig. 3 that the corresponding TO_{k3} tracks the input TI_{Vk} but with the constant error K_{VTO3}. According to eq. (15) K_{VTO3}=-6.25. After only eight steps, K_{VTO3} is about to reach the final value K_{VTO3}=-6.25, proving once more the correctness of all previous analysis.

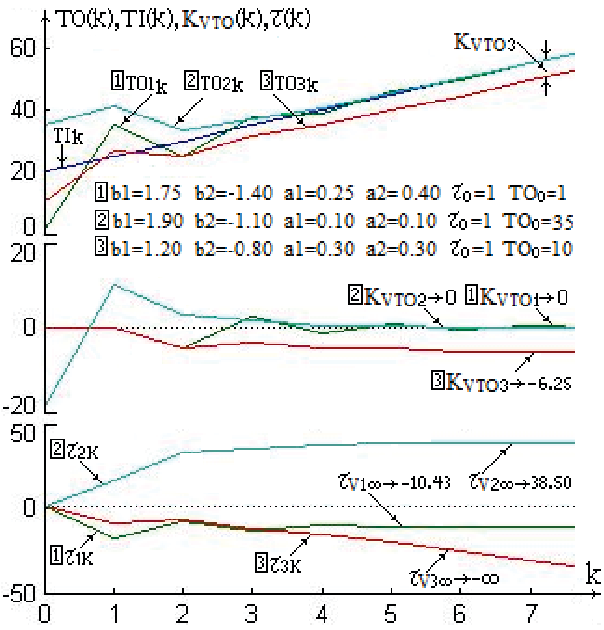

Fig. 3. Tracking of the input ramp function for three combinations of parameters.

The noise rejection ability of $TFLL_2$, in the function of system parameters, is demonstrated by simulation and shown in Fig. 4. The input period TI_k is the step of 10 t.u., which is strongly corrupted by the uniform distributed noise. The amplitude of noise is 9 t.u. peak to peak. Three outputs TO_{1k}, TO_{2k}, and TO_{3k} are presented in Fig. 4, depending on the different parameters for the same input. In case of TO_{1k} where b_1 and b_2 are very small, the output periods are completely cleaned from noise. For ten time higher b_1 and b_2, the periods of TO_{2k} are a little noisy. At last, for the large b_1 and b_2, the influence of the noisy input is the stronger in TO_{3k}. Note that, even for the worst case, noise in TO_{3k} is significantly reduced in comparison with the input noise. That means $TFLL_2$ is naturally suitable circuit for noise rejection applications. It can be concluded that if the sum of a_1 and a_2 is grater, the influence of noisy input is smaller. Another important conclusion is that a larger sum of b_1 and b_2, in Fig. 4, leads to a longer transition time of the $TFLL_2$. In other words, for the better noise rejection, the $TFLL_2$ becomes automatically slower, i.e. its transition time becomes longer.

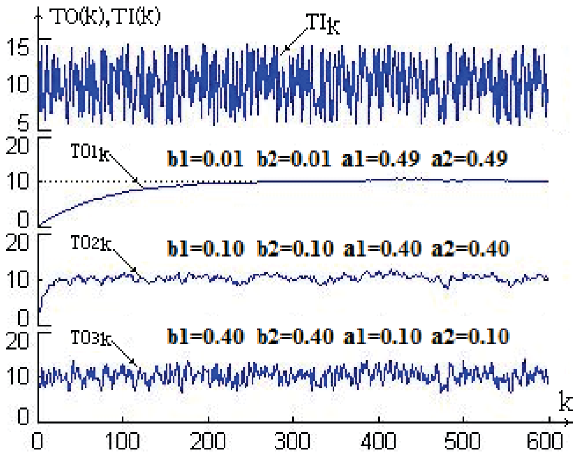

Fig. 4. *The input is strongly corrupted by noise. The smaller values of b_1 and b_2 provides better noise rejection and the longer transition time.*

DESIGN OF IIR TIME DIGITAL FILTER

References [1, 2] show how to design a FIR (Finite Impulse Response) Time digital filter intended for filtering the period of an impulse signal based on TFLL. For this purpose, the theory of classical digital filters is used, as well as software tools from Mat-lab intended for the analysis and design of digital filters. In this article, we will describe the process of developing an IIR (Infinite Impulse Response) Time second-order digital filter based on TFLL. Using the corresponding TFLL, let us design a simple digital low-pass Butterworth filter with next properties: cut-off frequency (3 dB), cutoff frequency f_c=2 kHz, minimum attenuation of 30 dB at stop band frequency, cutoff frequency of stop band f_b=4.25 kHz and the sampling frequency f_s=10 kHz. The first step is to design classical digital filter with the required properties. The transfer function H_{df} of the second-order digital filter, which satisfies the requirements, is presented in eq. (18). Note that b_{0d}=0.20657, b_{1d}=0.41315, b_{2d}=0.20657, a_{1d}=-0.36953 and a_{2d}=0.19582. The frequency response of this filter is presented in Fig. (5).

$$H_{df}(z) = \frac{0.20657 \cdot z^2 + 0.41315 \cdot z + 0.20657}{z^2 - 0.36953 \cdot z + 0.19582} \quad (18)$$

Let us now determine the corresponding TFLL which is able to cover all zeros and poles of the transfer function $H_{df}(z)$. All zeros and poles can cover $TFLL_3$ whose difference equation is shown in eq. (19). That is the third-order $TFLL_3$, but it is not full version. The part

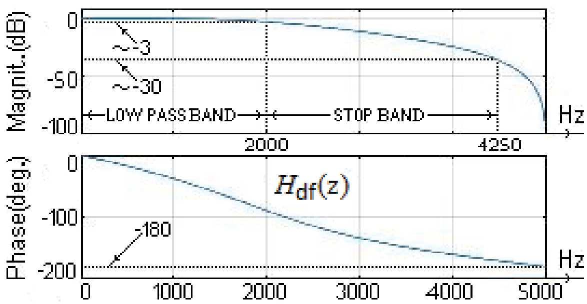

Fig. 5. *Frequency response of Butterworth digital low-pass filter satisfies all requirements.*

"$a_3 \cdot TO_k$" is missing from eq. (19). The transfer function of $TFLL_3$ is presented in eq. (20). We can see that the transfer functions $H_{TO3}(z)$ and $H_{df}(z)$ possess the same number of zeros and poles. If we now define $b_1=b_{0d}$=0.20657, $b_2=b_{1d}$=0.41315, $b_3=b_{2d}$= 0.20657, $a_1=-a_{1d}$=0.36953 and a_2=-a_{2d}=-0.19582, eq. (20) will turn into eq. (21). By comparing eqs. (18) and (21), we can see that the ratio of $H_{df}(z)$ and $H_{TO3}(z)$ is given in eq. (22). They differ only for factor "z^{-1}". It means that the transfer function $H_{TO3}(z)$ enters an additional delay of the input signal in comparison to the transfer function $H_{df}(z)$. This delay is 2π [rad] for the full range of the sampling rate, i.e. π [rad] for the half of the sampling rate. The frequency response of the $TFLL_3$ is presented in Fig. (6). We can see in

$$TO_{k+3} = b_1 TI_{k+2} + b_2 TI_{k+1} + b_3 TI_k + a_1 TO_{k+2} + a_2 TO_{k+1} \quad (19)$$

$$H_{TO3}(z) = \frac{TO_3(z)}{TI(z)} = \frac{z^2 b_1 + z b_2 + b_3}{z^2 - z a_1 - a_2} \cdot z^{-1} \quad (20)$$

$$H_{TO3}(z) = \frac{0.20657 z^2 + 0.41315 z + 0.20657}{z^2 - 0.36953 z + 0.19582} \cdot z^{-1} \quad (21)$$

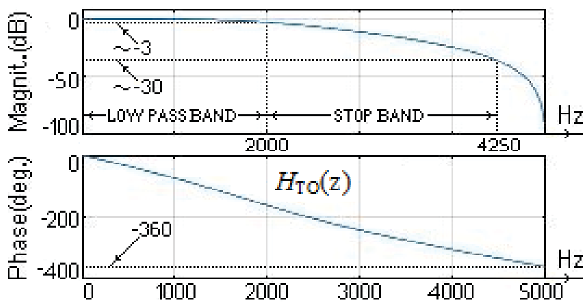

Fig. 6. *Frequency response of Butterworth Time digital low-pass filter based on $TFLL_3$.*

Figs. (5) and (6) that the magnitudes of digital filter and $TFLL_3$ are identical, but the phases differ for expected – π [rad] for the half range of the sampling rate. The requirements for Time digital filter based on $TFLL_3$ are fulfilled, proving the correctness of the previous analysis and presentations.

$$H_{TO3}(z) = H_{df}(z) \cdot z^{-1} \quad (22)$$

CONCLUSION

The described design of TFLL represent the further deepening to the recently described theory, design and application of TFLL, TPLL and Time digital filter, presented in refs. [1] to [12].

This $TFLL_2$ offers considerably wider possibility for the choice of the system parameters in comparison with similar TFLL of the first order, described in ref. [9]. In this respect $TFLL_2$ possesses better performances in the noise rejection applications. The advantages of this $TFLL_2$ are especially evident in the applications which require the trading of the extent of noise suppression and reduction of the system transient time. This $TFLL_2$ is also more powerful in the tracking applications in comparison to TFLL described in ref. [9]. Unlike TFLL in ref. [9] which is able to track the ramp input but with the constant error, this $TFLL_2$ provides the tracking of the ramp input without any error.

It is of interest to emphasize that TFLLs and the classical digital filters represent completely different types of systems. The first one is based on the processing of the periods of the impulse signals and time differences between them. In other word TFLLs process the time. The other ones are based on the processing of amplitudes. Regardless of that, the article showed that Mat-lab tools and the theory of IIR digital filters can be completely used for the analyzes and design of TFLLs in the frequency domain, as well as for the design of Time digital filters, based on TFLLs. In this work, it was shown practically how to understand the physical aspects of the TFLL processing, compared with the digital filter processing and how to identify the meanings of TFLL variables which we come across the usage of Mat-lab tools. Due to the mentioned contributions, Time Recursive Processing has got new weightiness and significance in the different scientific fields.

Acknowledgements

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